**CHAPTER 2 DRAFT #2 August 18, 2016**

**STATICS OF PARTICLES IN TWO DIMENSIONS**

**2.1 Introduction**

In Chapter 1, a force was defined as the action of one body on another. In simple terms, a force could be described as a “push” or “pull” on a body or object. Forces cannot be seen but the effects of a force can both be seen and felt.

If a car crashes into a stationary pole, the damage to both the car and the pole can be seen as the effect of the forces involved in bringing the car to rest. The car has exerted a force or “action” on the pole and the pole has exerted a force or “reaction” back on the car that brings the car to rest. The driver of the car and any passengers within the car unfortunately in turn feel the effect of the force that brings their forward motion to rest.

The force that one object exerts on another object may be a force concentrated at point. In t 2.1(a) concentrated forces are shown acting at points C and D of the beam. Forces may also be characterized as a series of forces distributed over the mass of an object as shown in Figure 2.1(b).

Distributed forces are often called distributed loads. Common examples of distributed loads that occur in nature include hydrostatic pressure, snow and wind loads. Distributed loads may be evenly or unevenly distributed. Distributed forces will be discussed in more detail in Chapter 3 when the statics of rigid bodies is discussed. In this chapter, a force applied to a **particle or at a point** will necessarily be a concentrated force,





1. (b)

**Figure 2.1**



**Photo 1**

In this chapter, forces acting on a particle are studied. A force is a vector quantity and as such has a magnitude and a direction. Vector analysis that is necessary for this introductory course in Statics is presented in Appendix A-5.

Vector quantities add according to the parallogram law. The parallogram law or the alternate triangle rule will be applied to replace two forces acting on a particle by a single force called the resultant of the two forces, **R**.

Both graphical methods and trigonometric methods using the sine and cosine rules are applied to solve a variety of problems where it is necessary to determine the resultant of any number of forces acting on a particle. Conversely, a force may be replaced or be resolved into components by again by applying the parallelogram law or the triangle rule.

The special case of resolution of a force vector into its rectangular components and the process of summing these components is then introduced as a more convenient method of determining the resultant force of any number of forces acting on a particle.

Chapter 2 concludes with the study of equilibrium of a particle under the action of forces. The concept of the Free-Body Diagram (FBD) used to represent all forces that act on the particle and the application of the equations of equilibrium to the FBD are introduced.

**2.2 Forces on a Particle**

**2.2.1. Characteristics of a Force**

In the previous section, it was said that forces are characterized by the **point of application**, the **magnitude** of the force and the **direction** of the force. Forces acting on particles have the same point of application, it is the particle itself. Therefore, a force acting on a particle is completely defined by stating its **magnitude** and **direction**.

**Magnitude** - In the SI system the Newton (N) and the kiloNewton (kN), 1 kN = 1000 N are used to specify the magnitude of a force. In the British and US customary system the pound or kilopound. (1000 lbs. = 1 kilopound or 1 kip) are used.

The magnitude of a force can be represented graphically by the length of a section of a line segment. In graphical solutions to problems of forces acting on a particle, a suitable scale either an “Engineers” or a “Metric” scale is first chosen. It is then used to represent the magnitude of each of the forces acting on the particle.

**Direction** - The direction of a force is defined by specifying;

a) the **line of action** of the force, and

b) the **sense** of the force

In two dimensional problems, these two terms are defined as follows:

**Line of Action**– The **line of action** of a force is an infinitely long straight line along which the force acts. To define the line of action of a force it is necessary to specify the angle that it makes with respect to a fixed reference axis such as a horizontal line (e.g. **the line of action** of a force is said to be 30o counterclockwise from a horizontal line). The force itself is a segment of the line of action. An appropriate scale is used to scale the length of the segment which is equal to the magnitude of the force. The force is shown as an arrow whose length is equal to the magnitude of the force.

**Sense** *–* The line of action of a force, however, only partly establishes the direction of a force. If for example a force is to be applied at point A which is fixed on a horizontal reference axis such that the line of action of the force makes a 30o angle measured counterclockwise from the fixed axis there is a dilemma. Is the force atA appliedso that it goes “upward to the right” or “downward to the left”? It is necessary to say or show which of the two possibilities is meant.

This is called stating the **sense** of the force. The **arrowhead** as illustratedin Figures 2.2(a) and 2.2(b) indicates the sense of a force. Note in Figure 2.2(b) that the arrow representing the force has a “tip” and a “tail”. Always refer to the arrowhead as the “tip” and the other end as the “tail”.



**Figure 2.2**

Force vectors will be represented by bold-faced letters e.g. (**F**). In longhand writing, as for example on the board or document projector, a vector may be denoted by drawing an arrow above the letter (****). The **magnitude** of a vector defines the length of vector and will be the same letter without an arrow above, e.g.  = = F. Magnitude of a vector is always a positive quantity.

**2.3 Addition of Vectors – Parallelogram Law and Triangle Rule**

All vectors add according to the Parallelogram Law. The Parallelogram Law lends itself to a graphical or approximate solution to problems involving two forces acting on a particle. The Triangle Rule is used as an alternate to the Parallelogram Law where a trigonometric or analytical solution to the problem is required.

**2.3.1. Resultant of Two Forces Acting on a Particle – The Parallelogram Law**

The **parallelogram law** is now introduced. The **parallelogram law** is the basis for **graphical** solutions to many statics problems involving forces acting on a particle.

## The Parallelogram Law

Two forces **P** and **Q** acting on a particle O as shown in Figure 2.3 may be replaced by a single force **R** called the resultant of the two forces by constructing a parallelogram using **P** and **Q** as two adjacent sides of the parallelogram.

**Graphical Solutions Using the Parallelogram Law:**

If the magnitude and direction of the two forces say **P** and **Q** acting on a particle are given, the parallelogram with can be drawn to a suitable scale. The scaled length of the diagonal of this parallelogram represents the magnitude of the resultant. A protractor may be used to measure the angle the diagonal makes with either the x-axis or the y-axis. This is sufficient to define the direction of the resultant.

**Summary: The scaled length of the diagonal of the parallelogram that passes through O represents the magnitude of the resultant, R and the angle  represents the direction.**

The Parallelogram Law is based on experimental evidence; it cannot be proved mathematically.

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**Figure 2.3**

**Summary of the Parallelogram Law for Two Forces (Graphical Solution)**

1. Select a suitable scale and draw the two force vectors **to scale** in the directions given. Do this by selecting an origin, O, on a reference line and measure the angles (directions) from this line. (The reference line for measuring angles is usually, but not always assumed to be horizontal.
2. Form the parallelogram by drawing parallel lines from the tips of the two force vectors.
3. Draw the diagonal and measure the diagonal length, and the angle the diagonal make with the reference line. This is the **magnitude** and **direction** of the resultant force vector.

**ACCURACY of Graphical Solutions:**

The accuracy of this **graphical solution** depends on the scale chosen and how carefully the magnitudes of the forces have been scaled and how carefully the angles are measured. Only rough estimates of magnitudes and directions can give a “ballpark” answer. Therefore the graphical method is an excellent way to quickly check a solution obtained by some other analytical method.

**Sample Problem 2.1 – Parallelogram Law**

Two forces act at point O as shown below. Using the parallelogram rule, determine graphically the magnitude and direction of the resultant, **R**, of the two forces.



Construct to a suitable scale, a parallelogram with P and Q as adjacent sides as shown.

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The measured scaled length of the diagonal of the parallelogram is the magnitude of R and the measured angle, is the direction of R with respect to the reference or established x-y axis system.

**2.3.2. The Triangle Rule as an Alternate to the Parallelogram Law**

An alternate method for determining the sum of two vectors can be derived from the **Parallelogram Law**. It is called the **Triangle Rule**. The terminology “triangle rule” is used to imply a graphical solution by drawing a triangle.

Consider two vectors added according to the **Parallelogram Law** as shown in Figure 2.4 below.



**Figure 2.4**

The side of the parallelogram opposite **Q** is equal in magnitude and direction to **Q**. Similarly the side of the parallelogram opposite **P** is equal in magnitude and direction to **P**.

Rather than drawing a parallelogram, ½ a parallelogram or a triangle can be drawn as shown in Figure 2.5.



**Figure 2.5**

Vector **Q** is moved and applied at the “tip” of **P**. The “tail” of **P** is connected to the “tip” of **Q**. A triangle is now formed as shown in Figure 2.5(a). Alternately, **P** could be applied at the “tip” of **Q** and the “tail” of **Q** connected to the “tip” of **P** as shown in Figure 2.5 (b) to form another triangle.

The sum of two vectors is therefore obtained by arranging **P** and **Q** in tip-to-tail fashion, and then connecting the tail of **P** with the tip of **Q** or alternatively, by arranging **Q** and **P** in tip-to-tail fashion, and then connecting the tail of **Q** with the tip of **P**.

Again, with care in scaling forces and measuring angles accurate results can be obtained. From Figures 2.5(a) and (b) it is concluded that **P** + **Q** = **Q** + **P**, i.e. the vector addition of vectors is commutative.

The **law of sines** or the **law of cosines** (trigonometric or analytic solution) can also be applied to the triangle that has been constructed to calculate the **magnitude** and **direction** of the **resultant** analytically.

**The Law of Sines (Sine Law):**

Consider an arbitrary triangle shown in Figure 2.6 with internal angles A, B and C. The length of the sides opposite the internal angles A, B and C are represented by a, b and c respectively.

The **Sine Law** expresses the trigonometric relationship between the sides and internal angles of a general triangle.

|  |  |
| --- | --- |
|  | Sine Law: |

**Figure 2.6**

**IMPORTANT:**

For any given triangle, if any two of the three sides and the internal angle opposite one of these two sides are known, the **Sine Law** can be used to determine the other three unknowns. (The **Sine Law** fails if only for example; a, b and c, or a, b and C are known.)

Alternatively, if any two angles and any one side are known all of the other unknowns can be determined. Remember that if two of the three internal angles the third angle the third angle can be calculated because the sum (Σ) of the internal angles of a triangle is equal to 180°.

**The Law of Cosines (Cosine Law):**

When the magnitude of any two of the three sides and the included angle between these two sides are known, the magnitude of the third side can be determined. Alternately, if the magnitudes of all three sides are known, the included angle can be determined (the angle between the two sides that are known).

**Cosine Law:**



**Two sides** **and included angle** known. Determine the third side. If an angle other than the included angle is known use the Sine Law.

**Three sides** of triangle known. Determine included angles using the Cosine Law.

**IMPORTANT:** When drawing the triangle to which the Sine Rule and/or Cosine Rule are to be applied it is recommended that it be drawn to a fairly exact scale. There are problems where more than one solution is possible and this may not be readily apparent unless care is taken in constructing the triangle.

**SAMPLE PROBLEM 2.2 – Triangle Rule as an Alternate to the Parallelogram Law**

Two forces act at point O as shown in Sample Problem 2.1 Using the triangle rule, determine graphically and analytically the magnitude and direction of the resultant, **R**, of the two forces.

Establish a coordinate system and construct to a suitable scale, a triangle with P and Q as adjacent sides as shown in Figure 2.6(a).



**Figure 2.6 (a)**

**Graphical Solution:** The resultant, R, joins the “Tail” of Q to the “Tip” of P. The measured scaled length of the side R of the triangle is the magnitude of R and the measured angle, is the direction of R with respect to the reference x-y axis system.

**Analytical Solution:** Using the triangle in Figure 2.6 (a), the internal angle,  may be calculated as  = 180o – 45o = 135o. Therefore two sides and the internal angle between the two sides are known. The Cosine Rule can be applied to determine the length of the third side which is the magnitude of R.



The direction of R must now be determined. The other two internal angles of the triangle are labelled  and  as shown in Figure 2.6(b). Using the calculated value of R = 4.2 kN, the Sine Rule is now applied to the triangle.



**Figure 2.6(b)**

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The angle q that R makes with the reference x-y axis system is:

 = 45o – 25.9o = 19.1o

Therefore:

**R** = 6.48 kN 

**2.3.3. Resultant of Several Concurrent Forces**

In this section it will be shown how the **Parallelogram Law** and the **Triangle Rule** can be used to determine the **resultant** of more than two forces in the same plane acting on a particle (acting at a point). Before beginning two new definitions are introduced:

**Definition: Coplanar** means acting in the same plane.

**Definition:** **Concurrent** means passing through the same point.

Consider a particleA acted on by several **concurrent**, **coplanar** forces **P**, **Q** and **S** as illustrated in Figure 2.7 (a). (The three forces are **coplanar** because they are all in the same plane.) They are **concurrent** because their lines of action all pass through the same point A.

In the previous section, it was shown that the sum or **resultant** of two forces (vectors) could be found by using the **Parallelogram Law** or the **Triangle Rule**.

The sum or **resultant** of three forces, say **P**, **Q**, and **S** can be obtained by first adding the force vectors **P** and **Q** and then adding the vector **S** to the vector **P** + **Q**. To add three vectors, the **Parallelogram** **Law** is applied two times as shown in Figure 2.7 (b).



**Figure 2.7 (a)**



**Figure 2.7 (b)**

The sum of any number of force vectors may be obtained by repeated application of the parallelogram law to successive pairs of force vectors until all the forces have been replaced by a single force or **resultant**.

Alternately, by the **Triangle Rule**, by repeated application of the **Triangle Rule** the same result is obtained. All of the given force vectors are arranged in **tip-to-tail** fashion.

The “tail” of the first force vector drawn is connected with the “tip” of the last force vector drawn. This vector represents the sum or resultant, **R**, of all of the given concurrent coplanar forces as seen in Figure 2.8.



**Figure 2.8**

Like the **Parallelogram Law**, the **triangle rule** may be extended to find the **resultant** of any number of concurrent coplanar forces. This is illustrated in Figure 2.9.

The **graphical solution** is a very powerful tool to find the resultant of any number of concurrent coplanar forces. Note that the manner of sequencing the vectors does not matter because of the vector’s commutative property under addition.



**Figure 2.9**

**2.4. Components of Forces**

**2.4.1. Resolution of Forces into Components**

In the previous sections we demonstrated that by using either the **Parallelogram Law** or the **Triangle Rule**, two or more forces acting on a particle could be replaced by a single force (a Resultant Force, **R**) which has the same effect on the particle. The reverse process is true:

**Components of a Force:**

A single force, **F** may be replaced by **two or more forces** that have the same effect on the particle. These forces are called the **components** of the original force, **F**.

**IMPORTANT:**

For any force **F**, there are an **infinite** number of possible sets of components.

**Problem Types involving resultant, R and two (2) component forces:**

Depending on what information is given in the problem statement, use either the Triangle Rule or Parallelogram Rule to solve the problem. The problem may be solved graphically (graphical solution) or analytically by trigonometry using the sine and/or cosine rules (trig solution).

There are two common recurring types of problems involving a resultant force, **R** and two component forces.

**Problem Type 1;**

From the problem statement it is known that two forces, **F**1 and **F**2 act on a particle. These forces can be replaced by their resultant, **R**.

**Given:**

(a) the magnitude and direction of the resultant, **R** of **F1** and **F2** and

(b)magnitude and direction of one of the two forces (or components) say **F1**

**Determine:** The magnitude and direction of the second force, **F2** that when added to **F1** will give the resultant, **R** of **F1** + **F2**.

The second component, **F2**, is found by applying the **triangle rule** as shown in Figure 2.10.

**STEP 1** – Select a origin (application point),measure angles and  (lines of action of **F1** and **R**) draw to a convenient scale the magnitudes of **F1** and **F2**.

**STEP 2** – From the “Tip” of **F1** draw **F2** to the “Tip” of R. Measure the length of F2 to determine the magnitude of **F2**. Measure the angle,  that **F2** makes with the horizontal to determine the direction of **F2**.

This same triangle may be used to apply the Cosine Rule and the Sine Rule to obtain the trigonometric or the analytical solution.



**Figure 2.10**

**Problem Type 2**

From the problem statement, it is known that two forces say **F**1 and **F**2 act on a particle.

**Given:**

(a) the magnitude and direction of the resultant, **R** of **F1** and **F2** and

(b)the lines of action (directions) of the two forces **F1** and **F2**

**Determine:** The magnitudes of the two forces **F1** and **F2**.

**STEP 1** – Plot to scale the resultant, **R** and the lines of action of **F1** and **F2** as shown in Figure 2.11(a).

**STEP 2** - Construct a parallelogram as shown in Figure 2.11(b) by drawing the line a-c starting at the “Tip” of **R** parallel to the line of action of **F1** and line a-b parallel to the line of action of **F2**.

The magnitudes of components **F1** and **F2** may be determined:

a) graphically by measuring the lengths of the sides of the parallelogram, or

b) by trigonometry by applying the law of sines to either of the triangles of the parallelogram with **R** as the diagonal.



**Figure 2.11**

**NOTE:** Other cases involving sets of two (2) components forming a resultant force, **R** may be encountered. In all cases, apply either the parallelogram rule or triangle rule depending on the information given.

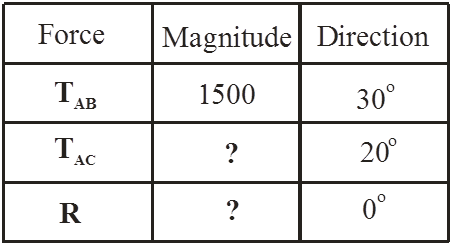
In some cases, the direction of one component may by given. It is also stated that the magnitude of the second component is a minimum but the direction is not specified. Problems of this type may be solved by **plotting to scale** the information given making sure that the length of the arrow representing the magnitude **minimum** force component is as small or short as possible in the diagram.

**Sample Problem 2.3**

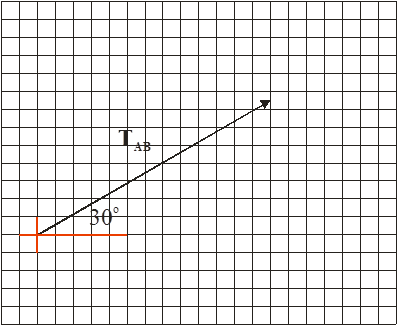
Two Tugboats are pulling a barge by means of two ropes. The tension in rope AB is 1500 N and the angle α is 20o. Knowing that the resultant of the two forces applied at A is directed along the longitudinal axis of the barge, determine by trigonometry:

1. the tension in the rope AC,
2. the magnitude of the resultant of the two forces applied at A.

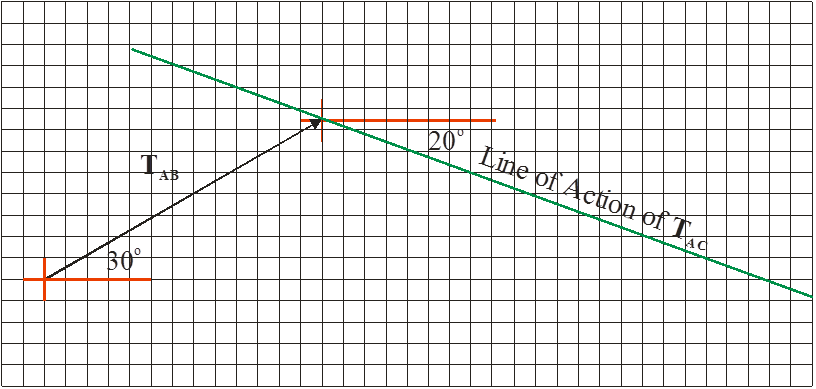




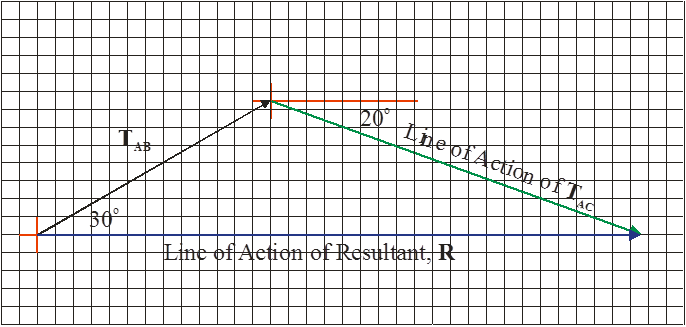
1. **Graphical Solution**

Begin by plotting TAB to scale since both its magnitude and direction are known.

At “Tip” of TAB Draw in Line of Action of TAC

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Draw in Line of Action of Resultant, R

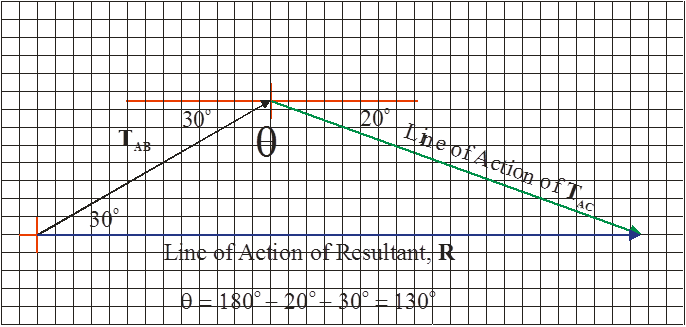
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**Scale Results:**

**R = 3200 N**

**TAC = 2000 N**

**b) Trig Solution**

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**Sample Problem 2.4**

The force **F** of magnitude *2000 N* in the Figure below is to be resolved into two components along line *a-a* and *b-b.* Knowing that the component of **F** along line *b-b* is to be *3407N* determine the angle α and the magnitude of the component along line *a-a*. Use:

1. A graphical method (parallelogram law or triangle rule), and
2. Trigonometry (sine and/or cosine laws).

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**Solution:**

**Step 1:** From the problem statement make an assessment of the type of problem. In this problem we are being asked to resolve a force, **F** into two components, (**Faa** and **Fbb**). A graphical solution as well as a trigonometry or analytical solution is asked for.

**Step 2:** Read the problem carefully and assess what information is given and what must be determined. It is **sometimes helpful** to construct a table where what is given and what is required is indicated in terms of the magnitudes and directions of the component forces and the resultant of the two components. (Here a question mark, **?**, is used to denote an unknown.)

|  |  |  |
| --- | --- | --- |
| **Force** | **Magnitude** | **Direction** |
| **F** | 2000 | 20o to **Fbb** |
| **Faa** | **?** | **?** |
| **Fbb** | 3407 | 20o to **F** |

**Step 3:** From the table, the magnitudes of force **F** and component **Fbb** of **F** are known as well as the angle that **F** makes with **Fbb** are known. It is known that when the two component force vectors, **Faa** and **Fbb** are placed “tip” to “tail”, the force **F** (**F** is the resultant of **Faa** and **Fbb**) drawn from the “tail” of the first component plotted to the “tip” of the second force component closes the triangle. This triangle may be drawn to scale and used to obtain a graphical or approximate solution. The sine and/or cosine rule may also be applied to obtain a trigonometric or analytical solution.

Start by plotting to scale what is known (or what is given), in this case plot **F** and **Fbb**.



**Graphical Solution:**

Draw a line b-b on grid paper. Select a suitable scale and scale off **F** = 2000 N at 20o to line b – b and **Fbb = 3407 N** along the line b-b.



“Tip” of **Fbb**

“Tip” of **F**

**Note:** At this point we only know Line a-a is at an angle, , to Line b-b.

**Graphical Solution:**

At the “Tip” of **Fbb** draw a line to the “Tip” of **F**.

Since **Faa** + **Fbb** = **F,** where **Faa** and **Fbb** are the components of **F**, the Line a-a must be parallel to this line. The segment of this line from the “Tip” of **Fbb** to the “Tip” of **F** is component **Faa** and can use our scale to determine its magnitude.

**Graphical Solution:**

From the completed component drawing, scale the length (magnitude) of **Faa** and

measure angle (the internal angle of the component triangle) with a protractor.



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**Trig Solution:**

**Trig Solution:**





**Sample Problem 2.5**

The hydraulic cylinder AC exerts a force, **P** on member BCD that is directed along the line AC. Knowing that the component of **P** parallel to CD is 1.35 kN, determine:

1. the magnitude of the force **P**, and
2. the component perpendicular to CD.





Establish a positive x-y coordinate system:

Make a table of what is known and what needs to be determined.

**P** is directed along the line AC.

Line AC is 60o to the horizontal and line CD is 55o to the line AC.

Therefore, CD is 60o – 55o = 5o to the horizontal.

|  |  |  |
| --- | --- | --- |
| **Force** | **Magnitude** | **Direction** |
| **P** | ? | 60o |
| **Component 90o to CD** | 1.35 kN | 90o to CD |
| **Component Parallel to CD** | ? | 5o |

**Graphical Solution:**

Select a suitable scale and establish a positive x-y coordinate system.

Draw the line CD at 5o and scale FCD = 1.35 kN

Construct a parallelogram with **P** as the diagonal:

* From point C, draw the line of action of the component of **P** perpendicular to CD or 60o to the horizontal.
* Draw the line of action of the component of **P** parallel to CD such that the
* ” tip” of this component meets the “tip: of P.

Measure to scale the magnitude of P and the magnitude of the component of P that is perpendicular to CD



**Trigonometric or Analytical Solution:**

Using either the upper or lower triangle created from the parallelogram with **P** as the diagonal:



**2.5. Rectangular Components of a Force**

In the previous section, the idea of resolving a given force **F** into **two** components was discussed. For any force **F** there is an infinite number of possible sets of components. While this concept is important, most engineering problems involving forces acting on particles do not involve the set of components that are oblique to each other and make it necessary to apply the Cosine and/or Sine Rule in the solution of the problem.

A two dimensional rectangular Cartesian coordinate system (x and y coordinate system) is now introduced and the set of two components ( **Fx** and **Fy**) that are **perpendicular** to each other are chosen as shown in Figure 2.12 below. The parallelogram is a **rectangle** and **F**x and **F**y are called the **rectangular components** of **F**.



**Fx** and **Fy** are called the rectangular components of **F**. They are perpendicular to each other. The parallelogram with **F** as the diagonal is a rectangle and the Pythagorean Theorem rather than the Law of Cosines may be used to determine values of the rectangular components.

**Figure 2.12**

This case, where a force **F** is replacedby two components that are **perpendicular** to each other is now examined. This is referred to as resolving the force into rectangular components. Resolving a force into its components is generally understood to mean resolving the force into its rectangular components

The force **F** is resolved into (see Figure 2.13)

**F**x - the vector component of **F** along the x axis

**F**y - the vector component of **F** along the y axis



**Figure 2.13**

**Orientation of the x and y axes:**

The orientation of the x and y axes is **usually** chosen, **horizontal and vertical**, however, there may be situations where it may be more convenient in the solution of the problem to change the orientation to other than horizontal and vertical as indicated in Figure 2.14.

In all problem solutions, the orientation of the x-y axis should be established and indicated as part of the solution.



**Figure 2.14**

**2.6. Representation of the Rectangular Components of a Force**

The rectangular components of a force are represented by stating the magnitude and direction of the each of the force components. The x-component will be in either the positive or negative x-direction while the y-component will be in either the positive or negative y-direction.

**2.6.1. Magnitude of the Rectangular Components of a Force**

Referring to Figures 2.12 and 2.14, the magnitudes of the x and y components of force **F** are the scalar values:

Fx = Fcosand

Fy = Fsin.

Fx and Fy are referred to as the scalar components or magnitudes of the rectangular components of **F**.

**2.6.2. Direction of the Rectangular Components of a Force**

In two dimensional or planar problems, the direction of the rectangular components may be represented in either Arrow Notation or Cartesian Vector Form or just Vector Form.

**Arrow Notation** - In arrow notation, an arrow indicating the direction is attached to the magnitude of the force component. The sense of the arrow is determined by inspection. In Arrow Notation, the **algebraic sign is dropped**. The sense of the arrow is sufficient to indicate a negative force component.

**Vector Notation (Form)** - In Cartesian vector notation or form, the unit vectors **i** and **j** are used to denote direction of the respective components**.** These unit vectors are directed along the **positive** *x* and *y* axis. The unit vectors **i** and **j** are dimensionless and note that 1(**i**) = **i** and 1(**j**) = **j**. Conversely , -**i** and –**j** would be directed along the negative x and negative y axes respectively.



**Figure 2.15**

The rectangular components **F**x and **F**ymay be found by multiplying the unit vectors **i** and **j** by scalar values of Fx and Fy, respectively.

That is,

|  |
| --- |
| **F** = Fx **i** + Fy **j** |

**F**x = Fx **i**

**F**y = Fy **j**

|  |  |
| --- | --- |
| **F**x and **F**y are referred to as the vector components of **F** and represent the magnitude and direction of the respective rectangular components. As with Arrow Notation, the sense of a force component in Vector Form is determined by inspection. However, the algebraic sign is retained to differentiate between a positive and negative component. |  |

The scalar component Fx is **positive** when the vector component **F**x has the same sense as the unit vector **i**. Similarly, the scalar component Fy is **positive** when the vector component **F**y has the same sense as the unit vector **j**.

The **unit vectors** attached to each scalar component give the **direction** of the component.

e.g., – 300**i** kN would be a force of magnitude 300 kN in the direction of the negative x axis.

**F** = 25**i** + 30**j** kN would be a force **F** with a 25 kN component in the positive xdirection and a 30 kN component in the positive y direction.

The **magnitude** of a rectangular component of the force **F**, shown in Figure 2.16 can be determined when the angle that **F** makes with the x axis (measured **counterclockwise** from the positive x axis) is given.

|  |  |  |
| --- | --- | --- |
| **Figure 2.16** | |  | | --- | | **Magnitudes of** **F**x **and** **F**y **:**  Fx = Fcosα  Fy = Fsinα | |

**2.6.3. Direction Cosines of a Force**

The magnitudes of the rectangular components of a force, **F**, may be expressed as the product of the magnitude of the force and the cosine of the angle that the force makes with each of the principal axes as shown in Figure 2.17..



Fx = Fcosθx

Fy = Fcosθy

The cosines of the angles θx and θy that the vector **F** makes with the x axis and the y axis are called the **Direction Cosines** of the vector.

The direction cosines of a vector are important when forces acting in three dimensions are studied.

**Figure 2.17**

**Sample Problem 2.6**

**Summary:**

**Rectangular Components of a Force in Two Dimensions:**

**F** = Fx **i** + Fy **j =** Fcosα **i** + Fsinα **j**

Fx and Fy are the **scalar** components of the force **F** and,

**i** and **j** are **unit** **vectors** in positive x and y directions.



A force of 500 kN is exerted on a nail as shown. Determine the horizontal and vertical components of the force and express them in vector notation.



**Solution:**

To solve this problem, first define or establish a positive coordinate system (sign convention):



By **inspection**, the scalar component Fx is positive since it has the same sense as the unit vector **i** and the scalar component Fy is negative since it has the opposite sense to the unit vector **j**.

Fx = + Fcosα = + 500 kN cos 60o = + 250 kN = Fcos300o

Fy = Fsinα = 500 kN sin 60o = 433 kN = Fsin300o

**Fx** = + 250**i** kN

**Fy** = 433**j** kN

Alternately, the components can be expressed using arrows (arrow notation) to denote direction:

**Fx** = 250 kN

**Important:** If arrow notation is used to indicate direction of a vector, the algebraic signs necessary for vector notation are dropped.

**Fy =** 433 kN

**2.7. Addition of Forces by Summing x and y (Rectangular) Components - RESULTANT FORCE**

Consider three forces **P**, **Q** and **S** acting on a particle A shown in Figure 2.18.



**Figure 2.18**

Their **resultant** **R** is:

**R** = **P** + **Q** + **S**

Rx**i** + Ry**j** = Px**i** + Py**j** + Qx**i** + Qy**j** + Sx**i** + Sy**j**

Rx = Px + Qx + Sx (Magnitude of the x component of the resultant) = 

Ry = Py + Qy + Sy (Magnitude of the y component of the resultant) = 

**Resultant of Forces:**

Rx = **Resultant** force x component is the sum of all the xcomponents of the forces.

Ry = **Resultant** force y component is the sum of all the y components of the forces.

Rx = ∑ Fx

Ry = ∑ Fy





and  is the angle that the resultant force R makes with x-axis.

**Sample Problem 2.7**

Four forces act on a bracket as shown in the figure. Resolve the four forces into their rectangular components and determine the magnitude of the resultant of the four forces.



|  |  |  |  |
| --- | --- | --- | --- |
| **Force** | **Magnitude**  (N) | **x – component**  (N) | **y – component**  (N) |
| F1 | 80 | -51.42 | +61.28 |
| F2 | 45 | -45 | 0 |
| F3 | 60 | -20.52 | -56.38 |
| F4 | 80 | 0 | -80 |
|  |  | Σ = -116.94 N | Σ = -75.51 N |



Show Resultant, **R**

either way

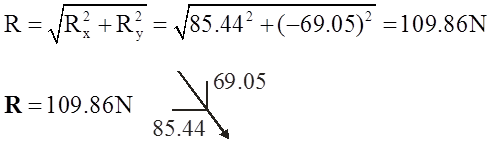


 **Sample Problem 2.8**

Determine the resultant of the forces shown.







 **2.7 Equilibrium of a Particle**



Sir Isaac Newton’s First law of Motion states that if the resultant of all forces acting on a particle is zero, the particle will remain at rest if it was originally at rest or will move in a straight line with a constant velocity if it was originally in motion. The particle is said to be in a state of static equilibrium.

**Equilibrium of a Particle:**

When the **resultant** of all forces on a particle is **zero**, the particle is in **equilibrium**.

###### **R** = **R**xi + **R**yj = 0 EQUILIBRIUM

therefore:

Rx = ∑Fx = 0

Ry = ∑Fy = 0

**2.7.1 Equations for Particle Equilibrium**

From the preceding discussion we have 2 equations to solve problems involving particles in two-dimensional (sometimes referred to as planar) equilibrium. Therefore, we can solve problems involving up to *two* unknowns.

Rx = ∑Fx = 0

Ry = ∑Fy = 0

These can be solved for 2 unknowns.

**2.8 The Free-Body Diagram of a Particle in Two Dimensional Problems**

Many problems involving planar equilibrium of a structure can be reduced to a problem of **equilibrium of a particle**. We do this by choosing a significant point (particle) in the structure and draw a separate sketch showing the particle and **all** forces acting on the particle. The sketch is a drawing of the particle isolated from its surrounding. This diagram is called the **Free-Body Diagram** (**FBD)** to which the equations of equilibrium are applied.

For the FBD, it is essential to:

1. Indicate and label all forces acting on the particle. (The labels used must match the labels when writing the equilibrium equations.
2. Indicate a sense for all of the forces. If the sense of a force acting on the particle is initially unknown, it may be assumed.
3. Indicate the angle or slope that each force makes with one of the principal axes. If the angle is unknown it should be indicated as an unknown in the FBD all forces acting on the particle MUST be labelled and the senses indicated.

**IMPORTANT: SOLVING EQUILIBRIUM PROBLEMS**

To solve equilibrium problems apply the equilibrium equations to the **FBD** of the particle.

It is therefore **absolutely essential** that the correct FBD is drawn for the particle under consideration. That is, **ALL** of the forces acting on the particle must be shown in the FBD (known forces and unknown forces).

If the sense of an unknown force is also unknown, **an assumption of the sense can be made**  when drawing the FBD.

When the equilibrium equations for the FBD for the FBD are solved, a negative answer for an unknown force will indicate that the initial assumption of the sense of the unknown force was **INCORRECT**. (A positive answer indicates a **CORRECT** assumption.) After solving the equilibrium equations for all unknowns the FBD should be redrawn showing the magnitude and correct direction of all forces acting on the particle.

**Sample Problem 2.9**

Two forces **P** and **Q** of magnitude 10 kN and 12 kN respectively are applied to the truss connection as shown. Knowing that the connection is in equilibrium and given the Free Body Diagram (FBD) as shown, determine the forces **T1**and **T2**.





**Sample Problem 2.10**

A welded connection is in equilibrium under the action of the four forces. Knowing that **P** = 30 kN and **FBA** = 25 kN and given the FBD shown, determine the magnitude of the other two forces. Redraw the FBD indicating the magnitude and correct direction of all forces.





**Note:** The senses of **FBD** and **FBC** in this FBD are **assumed (they may be incorrect).**

****



****

**Sample Problem 2.11**

A 800 N weight is suspended by two links AB and AC as shown. Determine the tension in the links AB and AC. Links AB and AC are in tension.



Note that the lines of action of the forces in links AB, AC and the force in the cable supporting the 800 N weight are all **“concurrent”** (pass through the same point) at A. We therefore choose point A as the point we want to isolate.

**STEP 1 – Selecting the “Significant Point” and Drawing the FBD**

Draw the **FBD** of the system we would like to examine. In this problem, the point A (point A is like a particle) can be isolated. That is, it is our **significant point.** In drawing the FBD of point A **we must show ALL** of the forces acting on point A. **(THIS IS CRUCIAL TO SOLVING THE PROBLEM!!!!!!).**

The two links support the weight and distribute it to the supports at pointsB and C. There are three forces acting at the point A; the known 800 N weight, the unknown tension in link AB and the unknown tension in link AC.

We draw the FBD of point A showing 800 N known gravity force acting downward and the two unknown tensions. (Imagine that you have isolated each of the links. To prevent the weight from falling you are applying a force in each of the links to hold the point A in its original position.)

We label the unknown tensions as **TAB** and **TAC**. (**TAB** means a tension force directed from pointAtoward point B. **TBA** would mean a tension force directed from point B toward point *A*).

We **ASSUME** the sense of the unknowns as directed **AWAY** from the point A, since the links are in tension.

We also indicate on the FBD **convenient** coordinate axes. (In this case, we have chosen a horizontal xaxis and a vertical yaxis.)

**Convenient** means that we can orient our xy coordinate system anyway that we want. For most statics problems, the x–axis will be horizontal to the page. However, in some problems you will see that it may be more convenient to orient the xaxis at some angle to the horizontal. The choice is yours.



We isolate point A and Draw a FBD of this point. We make an assumption as to the senses of forces **TAB** and **TAC** in our FBD.

Things to remember in drawing a FBD:

1. Label all forces and the point that you are drawing the FBD.
2. Indicate the magnitude of all angles.
3. Indicate the magnitude and sense of all known forces.
4. **Assume** a sense for all unknown forces.
5. Show what directions you are considering as positive.

**The FBD of point A is as follows:**

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**STEP 2**

Write the equilibrium equations for the point A.

Rx = ∑Fx = 0

Ry = ∑Fy = 0

TAB(cos45o) + TAC(cos30o) = 0 (1)

TAB(sin45o) + TAC(sin30o) – 800 = 0 (2)

**STEP 3**

We have 2 equations in two unknowns, TAB and TAC. Solve for the unknowns.

0.707TAB + 0.866TAC = 0 (1)

0.707TAB + 0.5TAC = 800 (2)

Adding (1) and (2): 1.366TAC = 800

TAC = +585.7 N

Substitute, TAC in (1):

0.707TAB + 0.866(585.7) = 0



Since our solution yielded **POSITIVE** answers for both **TAB** and **TAC** our assumptions as to the sense of the two unknown tensions in the FBD are correct.



**OPTIONAL HOMEWORK:**

Try this problem changing the sense of one of the unknowns and then both of the unknowns.

**STEP 4 (Optional but Recommended!)**

(a) Check by **graphical solution**



(c) Check by **trigonometry**



**Another good way to check is to express all sloping forces in terms of their horizontal and vertical components as shown below.**



**ΣFx = 0**

-507.13 +507.22 ≈ 0

**ΣFy = 0**

507.13 + 292.85 – 800 ≈ 0

**Sample Problem 2.11**

Two links are connected by a pin at B as shown in Figures (a) and (b). In each case, a 100 kN concentrated load is applied to the pin at B.

Draw a free-body diagram of point B for both examples, determine the force in links BA and BC and state whether the links are in compression or tension.





As a check, for Sample Problem 2.11 (a), redraw the FBD of the particle (Show all forces with their CORRECT directions) and resolve any sloping forces into their rectangular components and place them on the PLACEHOLDERS. The equilibrium equations are applied as a check!!!

**Part (a)** – Draw the FBD

**IMPORTANT**: In the **FBD** we assumed the senses of **TBA** and **TBD**. Because **TBA** and **TBD** were both positive our assumption of the senses is **CORRECT**!!!



As a check, for Sample Problem 2.11 (b), redraw the FBD of the particle (Show all forces with their CORRECT directions) and resolve any sloping forces into their rectangular components and place them on the PLACEHOLDERS. The equilibrium equations are applied as a check!!!

**IMPORTANT**: In the **FBD** we assumed the senses of **TBA** and **TBD**. Because **TBA** and **TBD** were both negative our assumption of the senses was **INCORRECT**!!!

**Part (b)** – Draw the FBD

**2.9 Forces in Ropes and Cables (Tension)**

Many particle equilibrium problems involving particles involve ropes or cables attached at a point. The ropes or cables support a weight(s). Since the ropes or cables must be in tension *(“you can’t push a rope”)* it is usually pretty easy to guess the correct **sense** of the **unknown** force in a rope or cable that is attached to a point.

The rope or cable will be “Pulling” on the point and the point will be pulling back with an equal force (Newton’s Third Law). Since points at either end are pulling on the cable, it is in tension.

**2.9.1 Representing the Sense of Unknown Forces in the FBD**

In representing an unknown force in our FBD, (in this case tension in a rope or cable), we may assume the sense of the tension force. As engineers, we should always indicate the correct sense of a tension force in a cable or rope in our FBD. However, as discussed previously, if for some reason, we assume the **wrong sense** of an unknown force when drawing the FBD, we will get a **NEGATIVE** answer when we solve the equilibrium equations for the FBD that we have drawn. After solving for all unknown forces, we simply redraw the FBD showing the magnitudes and correct directions of ALL forces. (It is important to check your calculations at this point by re-applying the equilibrium equations to the final FBD.)

**2.9.2 Representing the Sense of Known Forces in the FBD**

If we are **given** the magnitude of a tension in a cable or rope, we **MUST** show the correct sense of the given (known) force:

IMPORTANT:

**Drawing Given (Known Magnitude) of a Tension Force in the FBD**

Since a rope or a cable that carries a force is always in tension, if we are given the magnitude of the force in the rope or cable we MUST always show the sense of this force as acting AWAY from the point that we are drawing the FBD of.

* + 1. **Rope or Cable Passing over a Frictionless Pulley**

The tension in a rope or cable that passes over a frictionless pulley or through a frictionless ring remains constant. (It is assumed that the weight of the cable is small and can be neglected and that the cable does not stretch.)

As long as it is the **rope or cable is continuous**, it does not matter how many pulleys the rope passes over – **the tension in the rope or cable remains constant through the length of the cable. (Be careful if more than one rope is involved in a system of pulleys as the tensions in different ropes will be different.)**

Because ropes and cables are always in tension, these forces are always shown as “pulling” forces and are drawn in the direction of the cable as indicated in Figure 2.20.(a).

In Chapter 3, it will be proven that the tension force in a continuous cable that passes over a frictionless pulley remains constant.



**MULTIPLE PULLEYS** involving a **single continuous rope** or cable – rope or cable tension remains constant.

**Figure 2.20**

Referring to Figure 2.20(a), the equilibrium equation for moments (to be introduced in a later chapter) requires T1 = T2. Thus, for the cases in Figures 2.20(b) and (c):

T = W.

**MULTIPLE PULLEYS INVOLVING MORE THAN ONE CABLE:**

**NOTE:** There may be situations involving multiple pulleys where **different cables** are involved. The tensions in the different cables will not be the same and in a Free Body Diagram (FBD) of the pulley, these must be designated differently.

A rope or cable passing **through** (not attached) a smooth hook or ring also requires that T1 = T2 as shown in Figure 2.21.**.**



**Figure 2.21**

**Sample Problem 2.12**

A system of cables and weights as shown below is in equilibrium. Determine the tension in the cables and the magnitude of the weight W.



**STEP 1**

Draw the **FBD** of the system we would like to examine. We must keep in mind that cables can carry tension only. Examination of the statement of the problem and the associated diagram suggests there are two significant points that we could select to draw the FBD. In this problem we will draw the FBD of both points A and B.

We also indicate on the FBD **convenient** coordinate axes. (In this case, we have chosen a horizontal xaxis and a vertical yaxis.)



Note the labeling of the unknown forces in these FBD.

**STEP 2**

We can write two equilibrium equations for point A and two equilibrium equations for point B. Our solution strategy should be fairly obvious. For point A we have **two equations** and **two unknowns**. For point B, we have **two equations** and **three unknowns**. However, we know that once we have solved the equilibrium equations for point *A*, we will have determined **T**AB, which is also one of the unknowns at point B. Force **T**AB is equal in magnitude to **T**BA. The forces have the same line of action but opposite sense. (Again imagine that you are isolating points A and B by cutting through the cables. To prevent the weight from falling to the ground you are applying tension forces at the points where you have cut the cables. You are pulling upward to the right at point A and downward to the left at point B.)

**For point A:**

ΣFx = 0:

TAC(cos20o) + TAB(cos60o) = 0 (1)

ΣFy = 0

TAC(sin20o) + TAB(sin60o) –750 = 0 (2)

0.940TAC + 0.5TAB = 0 (1a)

0.342TAC + 0.866TAB – 750 = 0 (2a)

From (1a)

TAB = 1.88TAC

Substitute in (2a)

0.342TAC + 0.866(1.88TAC) – 750 = 0

1.97TAC = 750

TAC = 380.7 N

TAB = 1.88(380.7) = 715.7 N

**For point B:**

ΣFx = 0

TBD(cos20o) – TBA(cos60o) + TBE(cos45o) = 0 (1)

ΣFy = 0

TBD(sin20o) – TBA(sin60o) + TBE(sin45o) = 0 (2)

Substitute TAB = TBA = 715.7 N

0.940TBD – (715.7)(0.5) + (0.707)TBE = 0 (1a)

0.342TBD – (715.7)(0.866) +(0.707)TBE = 0 (2a)

0.940TBD + (0.707)TBE = 357.85 (1b)

0.342TBD + (0.707)TBE = 619.8 (2b)

Divide (1b) by 0.94, and (2b) by 0.342

TBD + (0.752)TBE = 380.7 (1c)

TBD + (2.067)TBE = 1812.3 (2c)

Adding (1c) and (2c);

2.819TBE = 2193

TBE = 777.9 N

TBD = 1812.3  (2.067)(777.9)

TBD = 204.4 N

TBE = Weight, W (Since cable passes through the ring.)

W = 777.9 N

The positive answers indicate our ASSUMPTIONS of the sense of the unknown forces in our FBD are CORRECT!!!!

**Sample Problem 2.13**

Two cables are joined at C and loaded as shown. Determine the tension in AC and BC.





****

Draw a FBD of point C where the tension cable forces and the weight forces are concurrent. **Convert the mass in kg to kN**. (Never mix mass and force in a FBD.) Calculate the hypotenuse of each of the slope triangles. Label all forces in the FBD. Establish a coordinate system for expressing the rectangular components of all forces in the FBD.

**FBD of Point C**

3000 kg = 3000(9.8) = 29,400 N = 29.4 kN

**IMPORTANT:** When writing the equilibrium equations for the FBD ALWAYS equate the left hand side of your equation to ZERO!!!! Make sure the labels in the equilibrium equations match the labels used for forces in the FBD.



The positive answers for TCA and TCB indicate the senses of these forces in the FBD was correct.

**Check: Redraw the FBD and place the components of the sloping forces on the “Placeholder” and apply the equilibrium equations!!!**

****



**Sample Problem 2.14**

A prehistoric spider of mass 7 kg is suspended from a portion of its web attached to two giant trees as shown in the figure. Assuming that the spider is in static equilibrium, determine the magnitude of the tension in strings AB, AE, BD and BC. String BC is horizontal.

**(Use g = 9.8m/sec2)**



**Free Body Diagrams**













**Check** – Resolve sloping forces into rectangular components ant put on placeholders: