STAT305, Winter 2004, Test 1 Time: 50 min.

1. Using only the axioms of probability, show that

   1) $P(A \cup B) = P(A) + P(B) - P(AB)$

   2) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)$.

2. A football club plays a match each week. If the club were to suffer four defeats in a row the rather impatient chairman would fire the manager immediately. Matches are independent of each other and the probability of defeat for any match is $\frac{1}{4}$. Last week the club managed to avoid defeat.

   (a) Let K be the remaining number of weeks the current manager will stay with the club. Find $P(K = k) for$ k= 4,5,6,7 *and* 8 .

(b) Find $E(K)$.

(c) In order to avoid getting fired, after the club suffers three consecutive defeats the unscrupulous manager bribes the referee of the next match (a bribe costs 25,000 dollars, gurantees avoiding defeat and the manager is never caught). What is the average amount per week the manager pays for bribes in the long run?

3. (i)For a branching process with family size distribution given by

$$P_0 = 1/6, P_1 = 1/2, P_3 = 1/3.$$

calculate the probability generating function of Z_2 given $Z_o = 1$, where Z_2 is the population of the second generation (simpliffy the expressions if time permits).

(ii) Find the mean and variance of the family size distrubution.

(iii) Find also, the mean and variance of Z_9 and the probability of extinction(where Z_9 is the population of the 9th generation). (Hint: Derive the mean and variance of the n the generation Z_n, clearly show all the steps).

4. A coin is tossed repeatedly, heads appearing with probability p on each toss.

(a) Let X be the number of tosses until the first occasion by which r heads have appeared successively. Write down a difference equation for $f(k) = P(X = k)$.

(b) Show the generating function of $f(k)$ is given by

$$F(s) = \frac{p^r s^r (1 - ps)}{1 - s + qp^r s^{r+1}}$$

(c) Find $E(X)$.