

STATISTICS 5.305 Final Exam

1. At time 0, there is a single cell. At each stage a cell either dies (probability $\frac{1}{6}$), divides into 3 (probability $\frac{1}{3}$) or remains unchanged.

(a) Show that the generating function of Y_{n+1} (of cells at time $n + 1$) is

$$G_{n+1}(t) = G_n\left(\frac{1}{6} + \frac{1}{3}t^3 + \frac{1}{2}t\right)$$

(Clearly show the details of the proof)

(b) Find the mean and the variance of the probability distribution of Y_n (ie. the number of cells at time $t=n$) (Clearly show the details of the proof).

(c) Calculate the probability of extinction π_0 .

2. Each customer who enters Rebecca's clothing store will purchase a suit with probability p . If the number of customers entering the store has a Poisson distribution with mean λ ,

(a) What is the probability that Rebecca sells at least one suit?(Show the details).

(b) What is the probability that Rebecca sells k suits?(Show the details).