

STATISTICS 5.305 Practice Problems

1. Let $\{B(t), t \geq 0\}$ be a Standard Brownian motion.
 - (i) Find $P(-4 \leq 2B(2) - 3B(5) \leq 6)$.
 - (ii) Find the variance of $B(2) - 3B(4) + 2B(7)$.
 - (iii) Find $Cov(3 + B(2) - 3B(4) + 2B(7), 9 + 3B(3) - 3B(5))$.
 - (iv) Find $E[3 + B(2) - 3(B(4))^2 + 2(B(7))^4]$.
 - (v) Find $P(-1 \leq B(2) \leq 3 | B(1) = 1)$.
 - (vi) Find $P(-2 \leq B(3) - B(4) \leq 1 | B(1) = 1)$.
 - (vii) Find $P(0 \leq B(3) \leq 4 | B(5) = 3)$.

2. Let $\{X(t), t \geq 0\}$ be a Brownian motion process with variance parameter $\sigma^2 = 3$.
 - (i) Find $P(-1 \leq X(2) \leq 5)$.
 - (ii) Find $P(-1 \leq X(3) - X(4) \leq 5)$.
 - (iii) Find $P(-1 \leq X(1) - 2X(2) + 3X(4) \leq 5)$.
 - (iv) Find $P(0 \leq X(4) - X(2) \leq 3 | X(7) - X(5) = 6)$.

3. Let $\{X(t), t \geq 0\}$ be a Brownian motion process with variance parameter $\sigma = 3$.
 - (i) What is the chance that at time 5, the process is less than 6?
 - (ii) If $X(7) = 6$, what is the chance that at time 12, the process is greater than 6?
 - (iii) If $X(12) = 10$, what is the chance that at time 5, the process is less than 6?
 - (iv) If $X(5) = 6$, what is the chance that at time 12, the process is greater than 10?
 - (v) Find $P(5.7 \leq X(2) \leq 6)$.

4. Let $\{B(t), t \geq 0\}$ be a Standard Brownian motion and let T_a be the first time that the Brownian motion hits a . For $t > 0$ let $M_t = \sup_{0 \leq s \leq t} B(s)$ be the maximum process over $[0, t]$.

Find the following

- (i) $P(M_4 \leq 5)$.
 - (ii) The density of M_4 .
 - (iii) The median of M_4 .
 - (iv) The first and the third quartile of M_4 .
 - (v) The mean and variance of M_4 .
 - (vi) The density and the median of T_2 .
 - (vii) The probabilities $P(T_2 \leq 4)$ and $P(2 \leq T_{-3} \leq 5)$.
 - (viii) The first and the third quartile of T_{-3} .
5. Suppose a person owns 1 share of stock currently worth 102 dollars. Assume that the change in value of this share over time follows a standard Brownian motion process where time is measured in months. What is the probability that the price three months from now is greater than 105?