STAT305, Winter 2005, Assignment 2 Due: Feb 7 th

1. A random number N of fair dice is thrown. $P(N = n) = 2^{-n}, n \geq 1$. Let S be the sum of the scores. Find the probability that

   a) $N = 2$ given $S = 4$

   b) $S = 4$ given $N = 2$.

   c) $S = 4$ given N is even

   d) the largest number shown by any die is r.

2. A biased coin ($p = P(head)$) is tossed repeatedly. Let P_n be the probability that an even number of heads has occurred after n tosses. Note that $P_o = 1$ and show that for $n \geq 1$,

$$P_n = p(1 - P_{n-1}) + (1 - p)P_{n-1}.$$

   Solve this difference equation to obtain an expression for P_n.

3. For a branching process with family size distribution given by

$$P_0 = 1/6, P_2 = 1/3, P_3 = 1/2;$$

   calculate the probability generating function of Z_2 given $Z_o = 1$, where Z_2 is the population of the second generation. Find also, the mean and variance of Z_2 and the probability of extinction. Repeat the same calculation when $Z_o = 3$ and

$$P_0 = 1/6, P_1 = 1/2, P_3 = 1/3.$$

4. (a) For a branching process, calculate π_0 when

   (i) $P_0 = \frac{1}{4}$, $P_2 = \frac{3}{4}$

   (ii) $P_0 = \frac{1}{4}$, $P_1 = \frac{1}{2}$, $P_2 = \frac{1}{4}$

   (iii) $P_0 = \frac{1}{6}$, $P_1 = \frac{1}{2}$, $P_3 = \frac{1}{3}$

   (b) Let the probability p_n that a family has exactly n children be αp^n when $n \geq 1$, and $p_0 = 1 - \alpha p(1 + p + p^2 + \cdots)$. Assume that all 2^n sex sequences in a family of n children have probability 2^{-n}. Show that for $k \geq 1$, the probability that a family has exactly k boys is $2\alpha p^k/(2 - p)^{k+1}$. Given that a family includes at least one boy, what is the probability that there are two or more boys?

5. Let X_1, X_2, X_3 be independent random variables taking values in the positive integers and having probability function given by $P(X_i = x) = (1 - p_i)p_i^{x-1}$ for $x = 1, 2, \ldots,$ and $i = 1, 2, 3$.

(a) Show that

$$P(X_1 < X_2 < X_3) = \frac{(1-p_1)(1-p_2)p_2p_3^2}{(1-p_2p_3)(1-p_1p_2p_3)}.$$

(b) Find $P(X_1 \leq X_2 \leq X_3)$.

6. (a) Derive the autocorrelation function of a moving average process of order 2.

   (b) Derive the autocorrelation function of a stationary autoregressive process of order 1.