STAT305, Winter 2004, Assignment 1 Due: Jan 21

1. Suppose that in an experiment, a fair die is rolled twice. Let A={the first outcome is even}, B={the total score is 4}, C= the total score, D=the absolute difference between two scores.

   (a) Which of A, B, C, D are events? Which of them are random variables?

   (b) Which of the following make sense? Which of them do not?

   (i) $A \cup B$, (ii) $P(C)$, (iii) $E(A)$, (iv) Var(D).

2. If X and Y are two random variables, what do we mean by

   (i) $F(x)$ is the cumulative distribution function of X?

   (ii) $X \leq 4$ is independent of $Y \geq 2$?

3. Using only the axioms of probability, show that

   1) $P(A \cup B) = P(A) + P(B) - P(AB)$

   2) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)$.

4. a) Prove that $P(ABC) = P(A|BC)P(B|C)P(C)$.

   b) Prove that if A and B are independent, then so are A^c and B^c.

5. Prove Boole's inequalities:

   (a) $P(\cup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$, (b) $P(\cap_{i=1}^{n} A_i) \geq 1 - \sum_{i=1}^{n} P(A_i^c)$.

6. Let $A_1 \supset A_2 \supset \cdots$ be a sequence of events. If $\bigcap_{i=1}^{\infty} A_i = \phi$(empty), show that

$$\lim_{n \to \infty} P(A_n) = 0.$$

7. A football club plays a match each week. If the club were to suffer three defeats in a row the rather impatient chairman would fire the manager immediately. Matches are independent of each other and the probability of defeat for any match is 0.2. Last week the club managed to avoid defeat.

   (a) Let K be the remaining number of weeks the current manager will stay with the club. Find $P(K = k) for$ k= 3, 4,5,6 and 7 .

(b) Find $E(K)$.

(c) In order to avoid getting fired, after the club suffers two consecutive defeats the unscrupulous manager bribes the referee of the next match (a bribe costs 20,0000 dollars, gurantees avoiding defeat and the manager is never caught). What is the average amount per week the manager pays for bribes in the long run?

8. A coin is tossed repeatedly, heads appearing with probability $p = 2/3$ on each toss.

(a) Let X be the number of tosses until the first occasion by which two heads have appeared successively. Write down a difference equation for $f(k) = P(X = k)$.

(b) Show the generating function of $f(k)$ is given by

$$F(s) = \frac{4}{27}s^2[\frac{2}{1-\frac{2}{3}s} + \frac{1}{1+\frac{1}{3}s}].$$

(c) Find an explicit expression for $f(k)$ and calculate $E(X)$.

9. A coin is tossed repeatedly, heads appearing with probability p on each toss.

(a) Let X be the number of tosses until the first occasion by which r heads have appeared successively. Write down a difference equation for $f(k) = P(X = k)$.

(b) Show the generating function of $f(k)$ is given by

$$F(s) = \frac{p^r s^r (1 - ps)}{1 - s + qp^r s^{r+1}}$$

(c) Find $E(X)$.