5.414 – 2004-2005
Assignment 1
Due: 24 Septmeber 2004
Prof. M. Samanta

1. Let $X_1, \ldots, X_n$ be iid random variables having pdf

$$f(x;\theta) = \mathrm{e}^{-(x-\theta)} I_{[X \geq \theta]}$$

where $\theta \in \Omega = (0, \infty)$. Let $T_{1n} = min(X_1, \ldots, X_n)$. Show that $T_{1n} \xrightarrow{p} \theta$ as $n \to \infty$. Write $T_{2n} = \bar{X} - 1$ where $\bar{X} = \sum_{i=1}^{n} X_i/n$. Is T_{2n} consistent for θ.

2. Refer to problem 1. Show that T_{1n} is a sufficient statistic for θ. *Hint*: Use Neyman's Factorization Theorem.

3. If T is sufficient for θ, then show that any one to one function of T is also sufficient for θ. (Use Factorization Theorem).

4. Suppose T is a non-negative statistic such that $E(T) = \theta$ and $Var(T) > 0$. Show that $\sqrt{T}$ is not unbiased for $\sqrt{\theta}$. *Hint*: Consider $Var(\sqrt{T})$.

5. $X_1, \ldots, X_n$ are iid rv's having the pdf

$$f(x;\mu) = \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-(X-\mu)^2/2},$$

for $-\infty < X < \infty$. Construct an unbiased estimator for μ^2 based on the sample mean $\bar{X}$. *Hint*: Calculate $E(\bar{X}^2)$.

6. Show by an example that every function of a sufficient statistic is not statistic.

7. Let X be a random variable having a binomial distribution with parameters n and p. The variance of X/n is $p(1-p)/n$. This is sometimes estimated by

$$T_n = \frac{\frac{X}{n}\left(1 - \frac{X}{n}\right)}{n}.$$

Is T_n an unbiased estimator of $p(1-p)/n$? If not, can you construct one by multiplying T_n by a constant?. (*Hint*: Calculate $E(T_n)$).

8. Refer to question 7. Use an appropriate theorem to show that $T_n^* = nT_n = \frac{X}{n}\left(1 - \frac{X}{n}\right)$ is a consistent estimator of $p(1-p)$.

9. Suppose $X_1, \ldots, X_n$ are iid rv's having pdf

$$f(x;\theta) = \frac{1}{\theta}\mathrm{e}^{-(x-\theta)} I_{[X>\theta]}.$$

Show that the statistic $T = (min(X_1, \ldots, X_n), \sum_{i=1}^n X_i)$ is sufficient for θ but not complete.