Applied Mathematics Letters ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Applied Mathematics Letters

journal homepage: www.elsevier.com/locate/aml

Recursive estimation for continuous time stochastic volatility models

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ARTICLE INFO

Article history:
Received 4 March 2009
Received in revised form 20 June 2009
Accepted 22 June 2009

Keywords:
Recursive estimation
Stochastic volatility
Ito's formula

ABSTRACT

Volatility plays an important role in portfolio management and option pricing. Recently, there has been a growing interest in modeling volatility of the observed process by nonlinear stochastic process [S.J. Taylor, Asset Price Dynamics, Volatility, and Prediction, Princeton University Press, 2005; H. Kawakatsu, Specification and estimation of discrete time quadratic stochastic volatility models, Journal of Empirical Finance 14 (2007) 424–442]. In [H. Gong, A. Thavaneswaran, J. Singh, Filtering for some time series models by using transformation, Math Scientist 33 (2008) 141–147], we have studied the recursive estimates for discrete time stochastic volatility models driven by normal errors. In this paper, we study the recursive estimates for various classes of continuous time nonlinear non-Gaussian stochastic volatility models used for option pricing in finance.

1. Introduction

In the last two decades, volatility models have received considerable attention with the emphasis being placed on state space models. From an econometric standpoint time-varying volatility models have been widely developed, recognizing the essence that the volatility and the correlation of assets change over time (see for example Heston and Nandi [1]). Although state space models in which the conditional mean of the observed process is modeled as stochastic process are useful in parameter estimation, it is widely recognized that *stochastic volatility models*, which model the volatility as a stochastic process [13,14], should be employed to estimate the volatility parameters.

A filtering procedure has been suggested for discrete time stochastic volatility models, for instance, see Gong et al. [2], and Kirby [3] with normal errors. For stochastic volatility models with time-varying parameters, when a new observation is coming in, a new volatility parameter is added, and hence it is almost impossible to estimate the time-varying volatility parameters. In order to construct an optimal recursive estimate for *non-normal stochastic volatility models*, we start with the following discrete time example.

Consider a nonlinear state space model given in Shiryaev [4]

$$\begin{aligned} \theta_{t+1} &= a\theta_t + (1+\theta_t)\eta_{t+1} \\ y_{t+1} &= A\theta_t + z_{t+1} \end{aligned} \tag{1.1}$$

where $z_t \overset{iid}{\sim} N(0, \sigma_z^2)$, $\eta_t \overset{iid}{\sim} N(0, \sigma_\eta^2)$, and the sequences $\{z_t\}$ and $\{\eta_t\}$ are independent, where $\{y_t\}$ process is observed and all θ_t is the parameter process. Then in Gong et al. [2], we have the estimate $\hat{\theta}_{t+1}$ and $\gamma_{t+1} = E[(\theta_{t+1} - \hat{\theta}_{t+1})^2 | F_{t+1}^y]$ as

$$\hat{\theta}_{t+1} = a\hat{\theta}_t + \frac{Aa\gamma_t}{A^2\gamma_t + \sigma_z^2}(y_{t+1} - A\hat{\theta}_t) \tag{1.2}$$

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doi:10.1016/j.aml.2009.06.014