UNIVERSITY OF MANITOBA

DATE: April 21 2012, 9 AM – 12 noon

FINAL EXAMINATION

TITLE PAGE

COURSE: MATH 1210

TIME: 3 hours

EXAMINATION: Classical and Linear Algebra

EXAMINER: Craigen/Klurman

FAMILY NAME: (Print in ink) ____________________

GIVEN NAME(S): (Print in ink) ____________________

STUDENT NUMBER: ____________

SIGNATURE: (in ink) ____________________

(I understand that cheating is a serious offense)

Please indicate your instructor by placing a check mark in the appropriate box below.

☐ MWF 9:30–10:20 O. Klurman ☐ MWF 1:30–1:20 R. Craigen

INSTRUCTIONS TO STUDENTS:

This is a 3 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 14 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left margin. The total value of all questions is 135 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

For full credit, answers in polar or exponential form must use the *principal argument*.

Question	Points	Score
1	45	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	135	

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1. **SHORT ANSWER QUESTIONS** (Get straight to the point—but where you are asked to show or explain something full credit will not be given for answers that do not display sufficient work to make your reasoning clear.)

[5] (a) If $X = A^2B^\top$ is a 2×3 matrix, what are the dimensions (what is the size) of B?

[5] (b) Find and simplify the value of $\cos(\theta)$, where θ is the angle between (i.e., determined by) the vectors $\mathbf{u} = [5, -2, 1]$ and $\mathbf{v} = [1, 2, 5]$

[5] (c) Determine whether the following two lines are parallel:

$$x = -1 + t, \quad y = 1 + 2t, \quad z = 1 + 3t$$

and

$$\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{9}.$$

[5] (d) For what value(s) of parameter a does the following system have infinitely many solutions?

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & a^2 - a \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ a - 1 \\ -3 \end{bmatrix}$$

[5] (e) Find all real values of a such that the following upper triangular matrix is NOT invertible:

$$\begin{bmatrix} a & 3 & 2 \\ 0 & a^2+1 & 3 \\ 0 & 0 & a^2-4 \end{bmatrix}$$

[5] (f) Write the following sum using sigma notation:

$$-\frac{3}{4}+\frac{5}{8}-\frac{7}{16}+\frac{9}{32}-\cdots-\frac{199}{2^{100}}.$$

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[5] (g) What is the remainder when $f(x) = (2x)^{2012} + 8ix^3 - 1$ divided by $g(x) = 2x - i$? (Simplify your answer.)

[5] (h) S is a set of five vectors in $\mathbb{R}^4$, whereas T is a set of four vectors in $\mathbb{R}^5$. One of the sets S and T must be linearly dependent. Which one, and why?

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[5] (i) Suppose $a, b, c, d \in \mathbb{R}$ and that $1 + i$, $2 - i$ are two of the zeros (over $\mathbb{C}$) of the polynomial $p(x) = x^4 + ax^3 + bx^2 + cx + d$. Find all the other zeros of $p(x)$. What is the value of d?

LONG/FULL ANSWER QUESTIONS

Show all necessary work for full credit

[10] 2. Find a vector equation, in parametric form, for the line of intersection of the following planes:

$$3x - 2y + 3z = -2, \qquad -x + y + 2z + 5 = 0.$$

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[10] 3. Let $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$ $C = \begin{pmatrix} -1 & 1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$. Observe that $|A| = 5$.
Further suppose that D is a 3×3 matrix such that $|D| = 3$. For each part either evaluate the expression or give a reason why it is not defined.

(a) $\det\left(2A^{-1}D^{\top}\right)$

(b) $B^2 + AA^{\top}$

(c) $B + C^{\top}A$

(d) $A - 2BC$

(e) $A + CB$

[10] 4. (a) Simplify and express in Cartesian form: $\dfrac{(\sqrt{3}+i)^{10}}{(\sqrt{3}-i)^{8}}$

(b) Find all real number(s) b such that $\dfrac{2+bi}{1-bi} = \dfrac{-7}{10} + \dfrac{9}{10}i$.

[10] 5. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by

$$T(\hat{\imath}) = \hat{\jmath} + \hat{k}, \quad T(\hat{\jmath}) = \hat{\imath} + \hat{k}, \quad T(\hat{k}) = \hat{\imath} + \hat{\jmath}.$$

(a) Find the matrix A associated with T.

(b) $\lambda = -1$ is an eigenvalue of the matrix A. Find all eigenvalues A.

(c) For each of the eigenvalues λ of A, find all corresponding eigenvectors $\mathbf{v}$.

[10] 6. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 4 & 0 & 2 \end{bmatrix}$.

(a) Find the characteristic polynomial of A.
(Do not attempt to find the eigenvalues of A.)

(b) Show that A DOES NOT have negative eigenvalues.

(c) A must have at least one (real) eigenvalue. Why?

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[10] 7. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$.

Use the Principle of Mathematical Induction to show that, for all $n \geq 1$,

$$A^n = \begin{bmatrix} 1 & 2^n - 1 \\ 0 & 2^n \end{bmatrix}.$$

[10] 8. Solve the following system by putting its augmented matrix into Reduced Row Echelon Form (RREF)

$$x - 2z = -4$$

$$x + y - 2z + 5 = 0$$

$$2x + 2y - 3z = -7$$

[10] 9. (a) Find the determinant of the following matrix by using elementary row (and/or column) operations. (Show your work. At least one row or column operation must be clearly used.)

$$A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & 0 & 2 \\ 3 & 6 & 1 & 4 \\ 4 & 9 & 0 & 5 \end{bmatrix}$$

(b) Is A invertible? Why, or why not?

(c) Find the entry of the adjoint matrix, $\mathrm{adj}(A)$, that is located in the second row and the third column.

[10] 10. Use Cramer's Rule to solve the following system of equations:

$$2x - 3y = 1,$$

$$x + 5y = 4.$$

(NOTE: A correct solution by any other method will not receive full credit!)