UNIVERSITY OF MANITOBA

DATE: April 17, 2014 FINAL EXAMINATION

TITLE PAGE

EXAMINATION: Techniques of Classical and Linear Algebra TIME: 150 minutes

COURSE: MATH 1210 EXAMINERS: Chipalkatti, Kucera, Moghaddam

FAMILY NAME: (Print in ink) ______________________________

GIVEN NAME: (Print in ink) ______________________________

STUDENT NUMBER: ______________

SIGNATURE: (in ink) ______________________________

(I understand that cheating is a serious offense.)

Please place a check mark (✓) in the box next to your section.

❑ A01 9:30–10:20 AM MWF (110 E2 EITC) G. I. Moghaddam

❑ A02 1:30–2:20 PM MWF (100 St. Paul's) J. Chipalkatti

❑ A03 9:30–10:20 AM MWF (100 St. Paul's) T. Kucera

INSTRUCTIONS TO STUDENTS:

This is a 150 minute exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. Moreover, no calculators, cellphones or electronic translators are permitted.

This exam has a title page, 10 pages of questions and also 1 blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left hand margin next to the statement of the question. The total value of all questions is 100 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	7	
2	7	
3	6	
4	8	
5	6	
6	7	
7	10	
8	8	
9	10	
10	15	
11	7	
12	9	
Total:	100	

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[7] 1. Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$. Use mathematical induction to prove the identity

$$A^n = \begin{pmatrix} 1 & 2n & n(2-n) \\ 0 & 1 & -n \\ 0 & 0 & 1 \end{pmatrix}$$

for all integers $n \geq 1$.

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[7] 2. Let z denote a complex number such that $(1+i)z = \overline{i-z}$. Find the Cartesian form of z.

[6] 3. Find all eigenvalues of the matrix $A = \begin{pmatrix} 3 & -3 \\ 3 & -1 \end{pmatrix}$.

[8] 4. Consider the polynomial equation $P(x) = 0$ where

$$P(x) = 3\,x^6 - 2x^5 + ax^4 + bx^3 - 7\,x^2 + 8\,x - 1\,,$$

where a and b are some nonzero real numbers. It is given that $P(x)$ has exactly three negative real roots.

(A) Determine which one of the following statements is true. (You must give adequate reasons for your answer.)

- a and b are both positive.
- a and b are both negative.
- a is positive and b is negative.
- a is negative and b is positive.

(B) Show that $P(x)$ must have at least one positive real root. (You do not need part (A) to answer this part.)

[6] 5. Find parametric equations for the line of intersection of the two planes

$$2x - 3y + 5z = 4 \qquad \text{and} \qquad 3x + 5y - 2z = 3\,.$$

[7] 6. Use Cramer's rule to find the value of y **only**. (No marks will be given for the use of any other method.)

$$\begin{aligned} x + y \quad +z &= 3 \\ -y \quad +3z &= 0 \\ 2x \qquad\quad -z &= 0 \end{aligned}$$

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[10] 7. Consider the linear system of equations

$$\begin{aligned} w - 3x \quad -y \quad +4z \quad &= 0 \\ -2w + 6x \quad -y \quad -2z \quad &= 0 \\ 3w - 9x \quad -5y \quad +16z \quad &= 0 \end{aligned}$$

First find the reduced row echelon form of the augmented matrix and then find all basic solutions of the system. Use your answer to find a solution in which $w = 1$ and $y = -1$.

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[8] 8. Consider the matrix $B = \begin{pmatrix} b & b & b \\ b & 2 & b+1 \\ b & b+1 & 2 \end{pmatrix}$ where b is any real number.

(A) Evaluate $|B|$.

(B) Find all values of b for which the matrix B is invertible.

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[10] 9. Show that the vectors $\mathbf{u} = \langle 2, -1, 3, -4 \rangle$, $\mathbf{v} = \langle 3, 1, -2, -1 \rangle$, and $\mathbf{w} = \langle 9, 8, -19, 7 \rangle$ are linearly dependent, and write $\mathbf{v}$ as a linear combination of $\mathbf{u}$ and $\mathbf{w}$.

10. Let $A = \begin{pmatrix} -2 & 0 & 1 \\ 1 & 1 & 0 \\ 3 & 0 & -1 \end{pmatrix}$.

[8] (a) Find A^{-1} by any method of your choice.

[3] (b) Use your answer from part (a) to solve the system $A^T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.

[4] (c) Find $\det\left(\frac{1}{4}\operatorname{adj}(A)\right)$. (Hint: you do not need to compute $\operatorname{adj}(A)$.)

11. Let T be the linear transformation from $\mathbf{R^2}$ to $\mathbf{R^2}$ defined by reflecting points about the line $y = x$.

[3] (a) Find the matrix of T.

[2] (b) Find a vector $\mathbf{v} = \langle v_1, v_2 \rangle$ such that $T(\mathbf{v}) = \langle \sqrt{2}\,, 3 \rangle$.

[2] (c) Find the matrix of T^{-1}.

12. Let $A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$. It is given that the characteristic equation of the matrix A is $-\lambda(\lambda - 3)^2 = 0$.

[6] (a) Find two linearly independent eigenvectors $\mathbf{u}$ and $\mathbf{v}$ corresponding to $\lambda = 3$.

[3] (b) It is given that $\mathbf{w} = \langle 1, 1, 1 \rangle$ is an eigenvector for the eigenvalue $\lambda = 0$. Explain why $\mathbf{u}$ and $\mathbf{v}$ are orthogonal to $\mathbf{w}$.

For rough work only; no work on this page will be marked.