The University of Manitoba

MATH 1210: Techniques of Classical and Linear Algebra (Winter Term 2018)

Final Examination
April 18, 2018

Time: 2 hours | Total Marks: 80

Last Name (IN CAPITAL LETTERS): ______________________

First Name (IN CAPITAL LETTERS): ______________________

Student Number: ______________________

Signature: ______________________

(I acknowledge that cheating is a serious offense.)

Place a check mark (✓) in the box corresponding to your section and instructor.

☐	Jaydeep Chipalkatti	M-W-F at 1:30pm in 205 Armes
☐	Tyrone Ghaswala	M-W-F at 9:30am in EITC E2 110
☐	Sergei Tsaturian	M-W-F at 9:30am in 100 St. Paul's

Instructions:
Please ensure that your paper has a total of 7 pages (including this page). Read the questions thoroughly and carefully before attempting them. You must **show your work** clearly in order to get any marks for your answers.

You are **not allowed** to use any of the following: calculators, notes, books, dictionaries or electronic communication devices (e.g., cell-phones or pagers).

You may use the back pages for continuing your work, but please indicate this clearly.

	Obtained	Maximum
Page 2		14
Page 3		13
Page 4		14
Page 5		12
Page 6		14
Page 7		13
Total		**80**

Q1. Find the sum [7]

$$\sum_{m=-1}^{8} (m+4)^2.$$

You may use the formulae

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \text{and} \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

Q2. It is given that $2-i$ is a root of the polynomial $f(x) = x^3 - x^2 - 7x + 15$. Find the other two roots. [7]

Q3. Consider the vectors [7]

$$\mathbf{u} = \langle 4, 1, 1 \rangle, \quad \mathbf{v} = \langle 3, a, -1 \rangle, \quad \text{and} \quad \mathbf{w} = \langle -2, 1, 7 \rangle.$$

If $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = 0$, find the value of a.

Q4. Let P be the plane in $\mathbb{R}^3$ defined by the equation $3x - 5y - 2z = 8$, and let L be the line with symmetric equations [6]

$$\frac{x-3}{4} = \frac{y-6}{2} = \frac{z+5}{1}.$$

Which of the following statements is true?

(1) L is entirely contained in P.
(2) L does not intersect P.
(3) L intersects P in a single point.

You must give adequate justification for your answer.

Q5. Solve the following system of equations by any method of your choice. [8]

$$x + y + z = -3, \quad -2x - y + z = 6, \quad y + 2z = 1.$$

Q6. Find all pairs of numbers (p, q) such that the matrix [6]

$$\begin{bmatrix} 3p + 2q & q - 2 & 1 \\ 0 & p + q - 1 & 0 \end{bmatrix}$$

is in reduced row-echelon form (RREF). Only the final answers will be marked, and not the work leading to them.

Answers: ______________________________

Q7. Consider the system of equations [6]

$$x + y + z = 2, \quad 5x - y - 3z = -4, \quad 2x - y - z = 1.$$

Find the value of y using Cramer's rule. No credit will be given for any other method.

Q8. Let $A = \begin{bmatrix} 3 & -5 & a \\ 2 & 0 & 1 \\ 2 & -3 & 7 \end{bmatrix}$. It is given that $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ is an eigenvector for A. [2+4]

(1) What is the corresponding eigenvalue?

(2) What is the value of a?

Q9. Consider the matrix [6+2]

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}.$$

(1) Use any method of your choice to find A^{-1}.
(2) Verify that $A\,A^{-1} = I_3$.

Q10. Consider the linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ given by the formula [2+4]

$$T(\langle v_1, v_2 \rangle) = \langle 6\,v_1 - 4\,v_2, -3\,v_1 + 2\,v_2 \rangle.$$

(1) Find $T(\langle -3, 7 \rangle)$.
(2) Find all vectors v (if any) such that $T(v) = \langle 2, 1 \rangle$.

Q11. Consider the vectors [7]

$$\mathbf{u} = \langle 2, -1, 1 \rangle, \quad \mathbf{v} = \langle 5, 0, -1 \rangle, \quad \mathbf{w} = \langle -5, -5, 8 \rangle.$$

in $\mathbb{R}^3$. Show that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent, and express $\mathbf{w}$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

Q12. Suppose that A is a symmetric 3×3 matrix with eigenvalues $4, 6$, and -7. Further suppose that $\langle 1, 2, -1 \rangle$ is an eigenvector with eigenvalue 4, and $\langle -1, 2, 3 \rangle$ is an eigenvector with eigenvalue 6. Write down an eigenvector with eigenvalue -7. You must give adequate justification for your answer. [6]