Rectangular powers and Ramsey theory

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ABSTRACT

For a finite structure $\boldsymbol{A}$ (e.g., a graph, poset, group, or lattice), let its set of finite powers be $\mathrm{Pow}(\boldsymbol{A}) = \{\, \boldsymbol{A}^n \mid n \geq 0 \,\}$ with $P_{m,n}(\boldsymbol{A})$ the set of all substructures of $\boldsymbol{A}^n$ isomorphic to $\boldsymbol{A}^m$.

Choose positive integers n, m, k, c with $n > m > k$. Then we call an onto map $\Delta\colon P_{k,n}(\boldsymbol{A}) \to [c] = \{1, \ldots, c\}$ a *c-colouring*. We seek $\boldsymbol{B} \in P_{m,n}(\boldsymbol{A})$ such that the restriction of Δ to $\{\, \boldsymbol{C} \in P_{k,m}(\boldsymbol{A}) \mid C \subseteq B \,\}$ is constant; such a $\boldsymbol{B}$ is said to be *monochromatic* with respect to Δ.

I will discuss the positive and negative results of this quest, couched in the language of rectangular powers and polymorphism clones.