MMMM 2016 Combinatorics Graduate Student Workshop

Abstracts

30 April to 1 May 2016

Andrii Arman, University of Manitoba

Properties of Fibonacci-sum graphs

For each positive integer n, the Fibonacci-sum graph G_n is the graph with vertex set $V(G_n) = \{1, 2, ..., n\}$ and edge set consisting of those pairs that sum to a Fibonacci number. Properties of G_n were investigated in 2014, when Fox, Kinnersley, McDonald, Orlow, and Puleo characterized those n for which G_n has a Hamiltonian path and described all such paths. In this talk I will show other properties of G_n, namely that G_n is planar, bipartite and that the automorphism group of G_n is of order at most 2.

Rob Craigen, University of Manitoba

Sylvester's Follies

It was the year that Canada was born.

Joseph James Sylvester had a thought: We know what ″orthogonal″ means for real matrices, but what about these monsters with complex numbers for entries? It seemed that there were a few competing ideas for the ″natural″ choice. Sylvester was a nomenclatur wizard – it was Sylvester who endowed many of the terms that are so familiar to us today in elementary matrix theory, including the word ″matrix″ itself.

Commenting on his extensive coinage of new mathematical terms from the mint of Greek and Latin, Sylvester referred to himself as the ′New Mathematical Adam′ —E. T. Bell

In Sylvester′s fertile mind was born a new idea: Inverse Orthogonal Matrices, one special case of a class of ″generalized orthogonalities″ that struck him as worthy of study, and he eagerly disseminated his thoughts.

Alas, some 26 years later it was to die an ignoble death at a deft blow of the rapier mind of the French master Jacques Hadamard, who rescued from the burning ruin a key object of interest, now known after him as Hadamard matrices, Sylvester′s contribution largely lost to memory.

Fast forward to the 21st Century. The adopted child of Hadamard has a rich ancestry and his associated conjecture reins still as one of the two great holy grails of combinatorics, though he himself could not have been called a combinatorist. And applications it has found, in data analysis, coding theory, filtering, spectroscopy, discrete transforms, design of experiments, and today, quantum computing.

In 1989 I published the second paper (since Sylvester) on inverse orthogonal matrices, to clean up the mess left behind. A few years later, these objects were raised, entirely independently, by algebraists asking a natural question which has now generated a sizeable small literature of its own.

We survey the ancient ruins of, and the fascinating menagerie of exotic creatures arising from, Sylvester's Folly.

Colin Desmarais, University of Mantioba

Lower and upper bounds on the maximum number of unit distances
For a given number n, what is the maximum number of unit distances between n points in the plane? In 1946, Erdős provided the first upper and lower bounds to this number, as well as upper and lower bounds for the case in three dimensions in 1960.

In this talk, I will outline the proofs for the lower bounds on the maximum number of unit distances, but I will focus on the upper bounds of these numbers. These results depend on results from geometric and extremal graph theory. I will also discuss open problems related to these numbers.

Bryan Freyberg, Michigan Technological University

Orientable distance magic graphs
If for a graph G there exists an orientation of the edges such that there is a directed distance magic labeling, we say that G is orientable distance magic. In this talk, we find orientable distance magic labelings of some products of graphs, namely the cartesian, direct, strong, and lexicographic products. We completely settle the existence for all of these products of two cycles except for some cases of the strong product. In addition, we show that even ordered hypercubes are orientable distance magic as well as give some general constructions for regular orientable distance magic graphs using these products.

Dalibor Froncek, University of Minnesota Duluth

Rosa-type labelings, graph decompositions and factorizations
We will present some of the well-known labeling techniques used for isomorphic decompositions and factorizations of complete graphs, as well as some newer, not-so-well-known methods. All of them stem from the original labelings introduced by Alex Rosa in 1967.

William Kocay, University of Manitoba

Constructing and coordinatizing n_3 configurations
An n_3 configuration consists of n points and n lines, such that every point lies on 3 lines, and every line contains 3 points. A configuration is called geometric if it can be realized by n distinct points and n distinct lines in the Euclidean plane. For example, Pappus's theorem of Euclidean geometry describes a geometric 9_3 configuration. A central question is to determine which n_3 configurations are geometric. Grünbaum has conjectured that every geometric n_3 configuration can be coordinatized with rational numbers in the plane. Methods of constructing and coordinatizing n_3 configurations are described.

Donald L. Kreher, Michigan Technological University

Finite abelian groups and sequenceability

Given a group G with identity I and a subset S of $G\backslash\{\text{I}\}$ we define the Cayley digraph $\overrightarrow{Cay}(G;S)$ by letting its vertices be the elements of G and having an arc (g_1, g_2) if and only if $g_2 = g_1 s$ for some $s \in S$. The set S is called the *connection set*. The complete directed Cayley graph on G has connection set $G \setminus \{I\}$.

An *orthogonal subgraph* of a directed Cayley graph on a group G with connection set S is a subgraph that contains for each $s \in S$ exactly one edge (x, y), with $x^{-1}y = s$.

An orthogonal directed Hamilton path of the complete directed Cayley graph is a permutation $g_0, g_1, g_2, \ldots, g_{n-1}$ such that the sequence of partial products $g_0, g_0g_1, g_0g_1, g_2, \ldots, g_0g_1g_2 \cdots g_{n-1}$ are distinct. Groups G having such a sequence are called *sequenceable* and were studied by Gordon in 1961.

In 1974 Gerhard Ringel asked when does there exists a permutation $g_1, g_2, \ldots, g_{n-1}$ of the non-identity elements of G such that the sequence $g_2g_1^{-1}, g_3g2^{-1}, \ldots, g_{n-1}g_{n-2}^{-1}, g_1g_{n-1}^{-1}$ is also a permutation of the non-identity elements. Groups admitting such a permutation are called *R-sequenceable* and are equivalent to the existence of an orthogonal directed $(n-1)$-cycle in the complete directed Cayley graph. Although Paige first studied these sequences in 1951 it was Ringel's map colouring problem for compact 2-dimensional manifolds that motivated the 1978 work of Friedlander, Gordon and Miller.

We complete the proof of the Friedlander, Gordon and Miller conjecture that every finite abelian group whose Sylow 2-subgroup either is trivial or both non-trivial and non-cyclic is R-sequenceable. This settles the question posed by Ringel for abelian groups.

In this presentation I will present our proof for odd order abelian groups and in a subsequent lecture Adrián Pastine will present the proof for even order Abelian groups.

This is joint work with Brian Alspach, and Adrián Pastine.

Bethany Kubik, University of Minnesota Duluth

A Game of Crowns

The generalized crown is a well-known family of bipartite graphs whose order dimension is given in terms of the parameters n and k. To each generalized crown one can associate a graph of critical pairs whose chromatic number is bounded above by the order dimension of the poset. In this talk we characterize the adjacency matrix of these graphs and through Mathematica can quickly produce an image of these graphs.

Olive Mbianda, University of Minnesota Duluth

Properly even harmonious labelling

A graph G with q edges is said to be properly even harmonious if the vertices of G can be labeled with the integers from 0 to $2q-2$ without repetition such that when the edge with vertex labels x and y is given, the label $x+y (\bmod 2q)$, the edge labels, are $0, 2, ..., 2q-2$. Recently two variants of harmonious labelings have been defined.

A function f is said to be an odd harmonious labeling of a graph G with q edges if f is an injection from the vertices of G to the integers from 0 to $2q-1$ such that the induced mapping $f(uv) = f(u) + f(v)$ from the edges of G to the odd integers between 1 to $2q-1$ is a bijection.

James McKeown, University of Miami (and UMD)

The combinatorics of the Waldspurger decomposition

In 2005 J. L. Waldspurger proved a remarkable theorem. Given a finite reflection group G, the closed cone over the positive roots is equal to the disjoint union of images of the open weight cone under the action of $1 - g$. When G is taken to be the symmetric group the decomposition is related to the familiar combinatorics of permutations but also has some surprising features. To see this, we give a nice combinatorial description of the decomposition.

Michael McKeown, University of Minnesota Duluth

Vertex-magic group edge labelling

A vertex-magic group edge labeling of a graph $G(V, E)$ with $|E| = q$ is an injection from E to an Abelian group Γ of order q such that the sum of labels of all incident edges of every vertex $x \in V$ is equal to the same element $\mu \in \Gamma$. We completely characterize all Cartesian products $C_n \Box C_m$ that admit a vertex-magic group edge labeling by $\mathbb{Z}_{2mn}$.

Debajyoti Mondal, Universtiy of Manitoba (CS)

On acyclic coloring of planar graph subdivisions

An acyclic k-coloring of a graph G is a mapping ϕ from the set of vertices of G to a set of k distinct colors such that no two adjacent vertices receive the same color and ϕ does not contain any bichromatic cycle. Every maximal planar graph with n vertices has a 1-subdivision that is acyclically 3-colorable (respectively, 4-colorable), where the number of division vertices is at most 2n (respectively, 1.5n). On the other hand, there exists a $1.28n$ (respectively, $0.3n$) lower bound on the number of division vertices for acyclic 3-colorings (respectively, 4-colorings) of maximal planar graphs. In this talk, we will briefly review the techniques used to achieve these results, and some related open problems.

Adrian Pastine, Michigan Technological University

R-sequenceable even ordered groups and orthogonal directed cycles

In his 1974 solution to the map colouring problem for all compact 2-dimensional manifolds except the sphere, Gerhard Ringel was led to the following group-theoretic problem: When can the non-identity elements of a group of order n be cyclically arranged in a sequence $g_0, g_1, g_2, ..., g_{n-1}$ such that the quotients $g_i^{-1}g_{i+1}$, $i = 0, 1, 2, \ldots, n$ (with subscripts modulo n) are all distinct?

The complete Cayley graph X on a group G is the complete directed graph where the edge (x, y) is labeled by $x^{-1}y$. The edges with a given label z in G form a 1-factor F_z and $\{F_z : z \in G\}$ is a 1-factorization of X. A subgraph H of X is an orthogonal subgraph if it contains exactly one edge of each of the one-factors. In this language Ringel's problem asks:

For which groups G does the complete Cayley graph X admit an orthogonal directed cycle?

In this continuation to Professor Kreher's talk, we will discuss R-Sequenceability of even ordered abelian groups.

Sergei Tsaturian, University of Manitoba

An introduction to flag algebras

Flag algebras is a concept in graph theory that was introduced by A.Razborov in 2007 and became quite popular in the last decade. With its help, a lot of results were proven, in particular about Turan densities and Ramsey multiplicity. The method allows to restate graph theory problems in algebraic language; many complicated combinatorial arguments become simpler and are generalized. The nature of theory also allows a wide use of computational devices. I will introduce this theory for a general audience and show its simplest applications by reproving some well-known results.

Yangsheng Zou, University of Manitoba (CS)

Intersection preserving mappings

Given two sets U and V with equal size, a bijection f from the k-sets of U to the k-sets of V is called an intersection preserving mapping, if for any two k-sets X and Y of U, $|X \cap Y| = |f(X) \cap f(Y)|$. Any bijection of U and V induces an intersection preserving mapping. We answer the question "is every intersection preserving mapping induced by a bijection of U and V" ?