05M-400

Reliability Assessment Using Discriminative Sampling and Metamodeling

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ABSTRACT

Reliability assessment is the foundation for reliability engineering and reliability-based design optimization. It has been a difficult task, however, to perform both accurate and efficient reliability assessment after decades of research. This work proposes an innovative method that deviates significantly from conventional methods. It applies a discriminative sampling strategy to directly generate more points close to or on the limit state. A sampling guidance function was developed for such a strategy. Due to the dense samples in the neighborhood of the limit state, a kriging model can be built which is especially accurate near the limit state. Based on the kriging model, reliability assessment can be performed. The proposed method is tested by using well-known problems in the literature. It is found that it can efficiently assess the reliability for problems of single failure region and has a good performance for problems of multiple failure regions. The features and limitations of the method are also discussed, along with the comparison with the importance sampling (IS) based assessment methods.

INTRODUCTION

One of the principal aims of engineering design is the assurance of system performance within given constraints. While uncertainties present in the real world, the satisfaction of constraints such as structural safety can be of primary concern. This leads to the development of reliability-based design. Reliability, as defined in Ref. (Ang and Tang 1984), is the probabilistic measure of assurance of performance. The assessment of reliability, or reliability analysis, is the foundation for the reliability-based design and the recent heated research topic reliability-based design optimization (RBDO).

As a general formulation, the performance function, or state function, is defined as an *n*-component system with design (variable) space $S_x \subseteq \Re^n$.

$$f(X) = f(x_1, x_2, ..., x_n) \tag{1}$$

Where $X = (x_1, x_2, ..., x_n)$ is defined on the space $S_x \subseteq \Re^n$ with a joint probability density function (JPDF) $j_x(x)$.

In reliability analysis, a limit state function, $g(X)$, is defined such that for a particular limit state value f_0 of the performance function,

$$g(X) = f(X) - f_0 = 0 \tag{2}$$

The limit state value, f_0, is equivalent to an engineering performance specification. This limit state function divides the variable space into the safe region for which $g(X) > 0$ and the failure region for which $g(X) \le 0$. Geometrically, the limit state Eq. 2 is an *n*-dimensional surface that is called the failure surface. One side of the failure surface is the safe region, whereas the other side of the failure surface is the failure region. The reliability of a system or product, *R*, is defined as the rate of compliance to the performance specification and is given by

$$R = P(g(X) > 0) = \int_{g(X)>0} \cdots \int j_x(x) dx \tag{3}$$

Eq. 3 is simply the volume integral of $j_x(x)$ over the success region. Conversely the failure probability of a system or product, P_f, is defined as the rate of noncompliance to the performance specification or the complimentary event of reliability and is given by

$$P_f = P(g(X) \le 0) = \int \dots \int_{g(X)\le 0} j_x(x)dx \tag{4}$$

Eq. 4 is simply the volume integral of $j_x(x)$ over the failure region. By definition $R + P_f = 1$. The quantitative evaluation of the true R or P_f often poses two major difficulties. One is the determination of the correct forms of $j_x(x)$, which is often unavailable or difficult to obtain in practice because of insufficient data (Ang and Tang 1975). Another is the determination of the limit state surface $g(X) = 0$, which separates the failure region and success region. When the computation of the performance function is expensive, such a difficulty aggravates. Therefore, direct calculation of reliability R or failure probability P_f from formula Eq. 3 or Eq. 4 may be impractical. There roughly exist two classes of methods to solve R or P_f.

The first class of methods are Monte Carlo Simulation (MCS) and its variations (Ang and Tang 1984). The MCS method involves three steps: (1) repetitive sampling from the set of random variables according to their respective probability distributions, (2) obtaining the corresponding performance function values to the samples, and (3) identifying whether failure has occurred. The estimated probability of failure is then simply the number of failures, m_f, divided by the total number of simulations, m, or $P_f = m_f / m$, when m is sufficiently large. MCS is accurate and reliable, especially for problems having multiple failure regions. Therefore, its results are often used as a standard to test other methods (Kloess et al. 2003). However, MCS is computationally intensive because it requires a large m. For implicit and computational intensive performance functions, the computation burden could become unbearable. Therefore, a class of more commonly-used method is based on mathematical analysis and approximation (Ang and Tang 1984; Wu and Wang 1998). These methods are based on the concepts of a "most probable point" or MPP. MPP was from the phrase "most probable failure point" coded in (Freudenthal 1956). For non-normal distribution problems, the original distribution was transformed into a standard normal distribution. The MPP, defined in the standard normal vector space U, is the most likely combination of random variable values for a specific performance or limit state value. By transforming the limit state function, $g(X)$, from the X-space to the U-space, the MPP is the point of minimum distance, d, from the origin to the limit state surface, $g(U) = 0$. d equals in value to the reliability index, β. By the Taylor expansion approximation on the MPP, the reliability R is set equal to $\Phi(\beta)$ in the standard normal distribution space. These approaches use approximate analytic techniques to alleviate the computational burden of direct MCS. These methods' computation efficiency is high, but for problems with a nonlinear limit-state function, their accuracy is doubtful (Ang and Tang 1984). They also have difficulties in solving problems of multiple failure regions.

More recently, the Importance Sampling (IS) method was used to combine the analysis method with sampling to improve the accuracy of analysis method (Wu 1994). Continuing on their previous work (Kloess et al. 2003; Zou et al. 2002), Zou and colleagues developed an indicator response surface based method, in which MCS is only performed in a reduced region around the limit state (Zou et al. 2003). Qu and Haftka (Qu and Haftka 2003) has proposed a probabilistic sufficiency factor when using MCS. As far as probabilistic design methods concern, though there are a large quantity of archived papers on this topic, most of them are based on the approximation of MPP (Du and Sudjianto 2003; Du et al. 2003; Liu et al. 2003).

As the engineering technologies advance, many engineering design problems involve expensive analysis and simulation processes such as finite element analysis (FEA) and computational fluid dynamics (CFD). Since FEA and CFD processes are based on complex and numerous simultaneous equations, these processes are often treated as "black-box" functions, for which only inputs and outputs are known. For reliability analysis problems involving expensive black-box functions, the computational burden aggravates, making the MCS method difficult. It also disables the traditional analysis methods unless response surface method is used as a surrogate for the performance function.

This work presents a new reliability analysis method, which has advantages of MCS and avoids disadvantages of both MCS and the MPP concept based analysis method. It does not rely on the conventional MPP, but re-defines MPP according to its literal meaning in the X-space and automatically searches for it. The proposed method is found efficient, accurate, and robust for reliability analysis in small scale problems. Section 2 will discuss the concepts and algorithm of the proposed method. Section 3 gives numerical examples and illustrations for problems of both single and multiple failure regions. Section 4 discusses features of the method and compares it with importance sampling based assessment methods. Section 5 gives the conclusions and future work.

PROPOSED METHOD

For black-box functions, metamodeling techniques are usually used (Haftka et al. 1998). A common practice is to obtain an accurate metamodel, or surrogate, to a black-box function. Based on the metamodel, optimization or sensitivity analysis is then performed. As we know, in general the more sample points are used to construct a metamodel, the higher fidelity would the model achieve. The difficulties for metamodeling-based approaches include the unsure loyalty of the model to the real function, and the exponentially increasing computation burden for high-dimensional problems. For reliability analysis, the model accuracy is important around the limit state. The accuracy, however, is of much less concern in either the success or failure region, as long as the model predicts $g(X) \leq 0$ or $g(X) > 0$ correctly. Such an observation indicates unevenly distributed sample points might be possible to construct a metamodel for the purpose of reliability analysis. Therefore a discriminative sampling strategy might be developed, so that more sample points are generated near or on the limit state while fewer points are in other areas. This is exactly the aim of this work.

KRIGING MODEL

The metamodel chosen in this work is the Kriging model as defined below (Jones et al. 1998; Martin and Simpson 2003; Wang and Simpson 2004).

$$\hat{y}(X) = \sum_{i=1}^{k} \beta_i f_i(X) + z(X) \tag{5}$$

Kriging model consists of two parts. The first part is a simple linear regression of the data. The second part is a random process. The coefficients, β_i, are regression parameters. $f_i(X)$ is the regression model. The random process $z(X)$ is assumed to have mean zero and covariance, $v(x_1, x_2) = \sigma^2 R(x_2 - x_1)$. The process variance is given by σ^2 and its standard deviation is σ. The smoothness of the model, the influence of other nearby points, and differentiability of the response surface are controlled by the spatial correlation function, R(.). Kriging is flexible to approximate different and complex response functions. The response surface of Kriging model interpolates sample points, and the influence of other nearby points is controlled by the spatial correlation function. These features are useful for the proposed reliability analysis method.

A Kriging toolbox is given by (Lophaven et al. 2002). It provides regression models with polynomials of orders 0, 1, and 2, as well as 7 spatial correlation functions for selection. This work uses the regression model with polynomials of order 0, and the Gaussian correlation model. A detailed description of Kriging is in the lead author's previous work (Wang and Simpson 2004).

SAMPLING METHOD

As we know, there exists many methods in Statistics to generate a sample from a given probability density function (PDF) (Ross 2002). These methods include inverse transformation, acceptance-rejection technique, Markov Chain Monte Carlo (MCMC), importance sampling, and so on. A recently developed method is especially capable for high dimensional problems and for problems with multiple modes (Fu and Wang 2002). The goal of such sampling is to generate sample points that conform to a given PDF, i.e. more points in the area that has high probability and fewer points in the area that has low probability, as defined by the PDF. Inspired by such sampling, the authors developed a Mode Pursuing Sampling (MPS) method before (Wang et al. 2004) for global optimization, which adapts the Fu and Wang's method (Fu and Wang 2002). MPS used a variation of the objective function to act as a PDF so that more points are generated in areas leading to lower objective function values and fewer points in other areas. It is thus in essence a discriminative sampling method. By generalizing the idea in (Wang et al. 2004), the authors defined a sampling guidance function, which could be developed according to the sampling goal to realize a certain discriminative sampling scheme (Shan and Wang 2004). A sampling guidance function of a random variable vector X is described as a function $\hat{z}(X)$ which satisfies:

i. $\hat{z}(X) \geq 0$
ii. representing the nature of the problem
iii. reflecting one's sampling goal, and
iv. expressing prior information if used iteratively

In this work, the sampling goal is to have more sample points near or on the limit state and fewer in other areas. A sampling guidance function, $\hat{z}(x)$, is defined as:

$$\hat{z}(X) = C_0 - |g(X)| = C_0 - |f(X) - f_0|, \; C_0 \geq |g(X)| \tag{6}$$

where, C_0 is a constant. It is easy to see $\hat{z}(X)$ is always positive. Viewing the $\hat{z}(X)$ function as a PDF, it means that areas close to the limit state have a high sampling probability. The further away from the limit state, the less likely to be sampled, guided by the $\hat{z}(X)$ function defined in Eq. 6. This property represents the problem and reflects our goal of sampling as discussed before. As one can see, since the performance function f(X) is to be approximated by the metamodel, which iteratively improves with more sample points, Requirement (iv) is thus also satisfied.

RE-DEFINING THE MOST PROBABLE POINT (MPP)

As described in Section 1, the MPP was defined in the standard normal vector space U as the most likely combination of random variable values for a specific performance or limit state value. Such a definition was given to facilitate the computation of the safety index. There are two problems associated with such definition. The first is because when transforming design variables of any distribution (X-space) to a normal distribution (U-space), approximation usually has to be made. It means that the MPP in the U-space is not guaranteed to translate to the point having the largest failure probability in the original X-space. Second, the conventional reliability assessment method based on the reliability index could be either conservative or risky as it is based only on the tangent line of the limit state (Ang and Tang

1984). This work re-defines MPP in its original space, and the proposed algorithm will search the redefined MPP directly in the original space and thus no transformation is involved.

Definition: *The most probable point (MPP) is the point in the original design space (X-space) having the largest joint probability density function (JPDF) value among all the points in the failure region.*

It is believed that the above definition restores the literal meaning of MPP, eliminates possible sources of error in transformation, and brings the search attention back to the original space, from which the optimal design will be eventually determined. This work will use MPP according to the above definition in the ensuing sections unless otherwise indicated.

THE ALGORITHM

This section provides a step-by-step description of the proposed procedure with the convergence criteria in a separate sub-section. The flowchart of the procedure is illustrated in Figure 1.

Step 1: Generate a large number of sample points in the design space according to the PDF of each random variable. From these sample points, $(2n + 1)$ initial points are chosen; n is the number of random variables. The end points along each variable direction, as well as the point defined by the mean of all x_i components, are chosen as the initial points. Note the number of initial points is modest and does not increase exponentially with the number of variables.

Step 2: Evaluate the initial points by calling the expensive performance function, $f(X)$. All of the points evaluated by $f(X)$ are referred as expensive points in this work.

Step 3: Construct a kriging metamodel based on the initial points. The kriging model is thus an approximation of $f(X)$.

Step 4: Randomly generate a large number of cheap points from the kriging model (e.g. 10^4). From these cheap points, new sample points are to be picked and be evaluated. We would like to avoid points that have an extremely low probability. Also we need evenly distributed sample points from which points of desired property defined by the sampling guidance function will be picked. The questions at this step are then 1) *how to determine the sampling region?*, and 2) *how to avoid extremely low probability points?* These questions are addressed by the following sub-algorithm:

- A large number of sample points as per given variable distribution have been generated at Step 1. From these points, one can identify the minimum and maximum value along each x component direction. These values define a hyper-box from which evenly distributed grid points can be generated and their JPDF values can be computed. Similarly, the JPDF values for all the sample points generated at Step 1 can be computed; and the minimum JPDF value, min(JPDF), can be obtained.

- Grid points whose JPDF value is less than min(JPDF) are discarded from the point set. As a result, the left-over grid points will be evenly distributed and all have a higher JPDF value than min (JPDF).

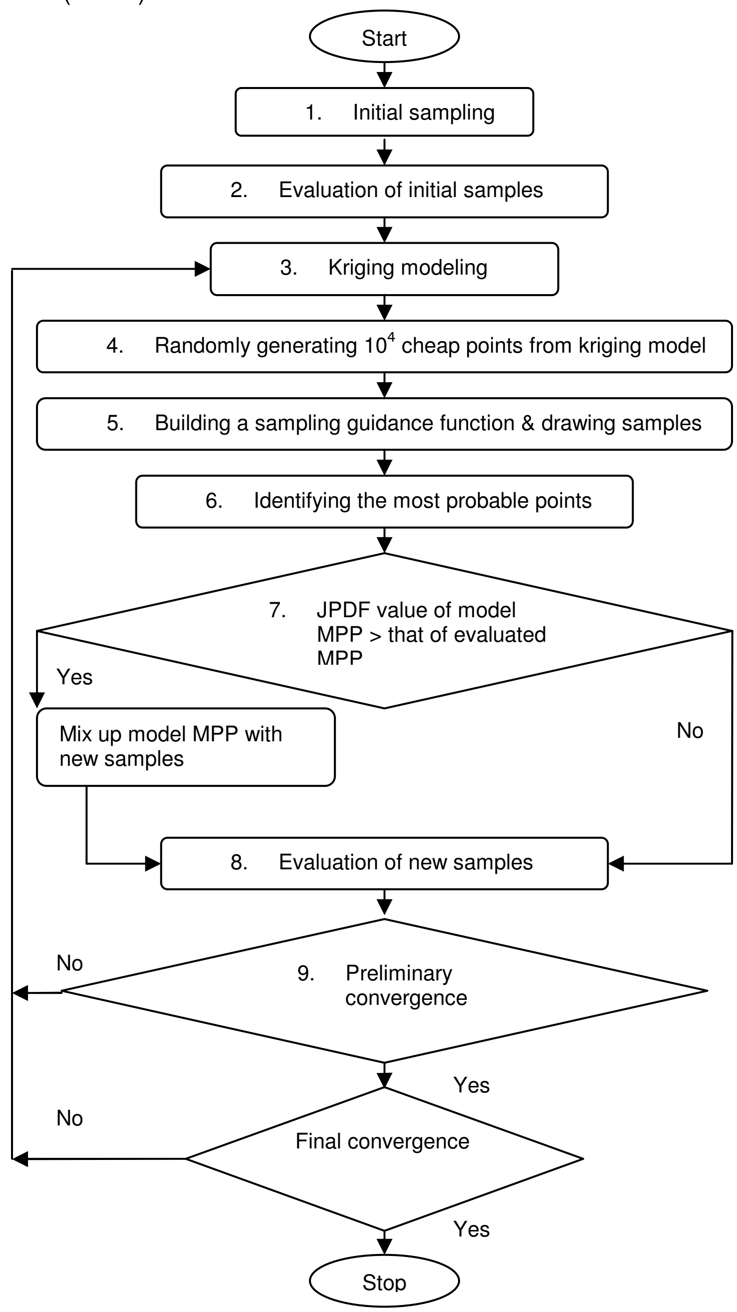

Figure 1 Flowchart of the proposed method.

Step 5: Build the sampling guidance function defined in Eq. 6. Draw n samples according to the sampling guidance function from the cheap points generated at Step 4.

Step 6: Identify the most probable point (MPP) and its corresponding JPDF value from the existing set of expensive points and the set of cheap points, respectively. MPP follows the definition given in Section 2.3.

Step 7: If the JPDF value of the most probable point obtained from the set of cheap points (model MPP) is greater than that of the point obtained from the set of expensive points (evaluated MPP), add the model MPP to the new sample set drawn at Step 5.

Step 8: Evaluate the new sample set.

Step 9: If convergence criteria are satisfied, the process terminates. Otherwise, go back to Step 3.

CONVERGENCE CRITERIA

This work applies two convergence criteria sequentially to obtain both the MPP and the reliability. The first criterion is when the model MPP is sufficiently close to the evaluated MPP. This criterion is realized by two conditions:

1. *abs* (JPDF value of model MPP– JPDF value of evaluated MPP) <= ε_1, and
2. *max* {x_i of model MPP – x_i of evaluated MPP), i =1,.., n} <= ε_2 (7)

where both ε_1 and ε_2 are small numbers. ε_1 is usually taken as 0.1; ε_2 is roughly one tenth of the converged coordinate value.

The second criterion is when the estimated failure probabilities in two consecutive iterations are sufficiently close. After each iteration, we will have an updated kriging model of the performance function. One can then apply the MCS method on the kriging model to estimate the reliability. In this work, 10^4 cheap points are used in MCS. The second criterion can then be expressed as

$$|P_{f,k} - P_{f,k-1}| < \varepsilon_3 \quad (8)$$

ε_3 is a small number. In this work, ε_3 is a constant, 3e-4. Since the check of the second criterion is comparatively more computationally intensive than the first criterion, the second criterion will be applied only when the first criterion is satisfied. Therefore, the two criteria defined by Eqs. 7 and 8 are applied sequentially, shown as Preliminary convergence and Final convergence in Figure 1.

NUMERICAL EXAMPLES

Two numerical examples are taken from the literature for testing the proposed method.

EXAMPLE 1

Example 1 (Zou et al. 2002) has a performance function $f(x_1, x_2) = x_1^3 + x_2^3$. Its limit state function is $g(x_1, x_2) = x_1^3 + x_2^3 - 18$. The distribution of random variables is $x_1 \sim N(10,5), x_2 \sim N(9.9,5)$.

Since there are random processes involved in the proposed method, 10 independent test runs have been carried out and their results are listed in Table 1 with $\varepsilon_1 = 0.1; \varepsilon_2 = 0.2$. The JPDF values of obtained MPP are listed in the second and fourth column; their coordinates are in the neighboring column, respectively. The 6th column lists the number of total iterations, which indicates the potential for parallel computation. The total number of performance function evaluations is in the 7th column. The 8th column lists the number of misjudged points (failure or success) by the kriging model as checked against the real performance function, for a total of 10,000 MC sample points. As one can see from Table 1, the kriging model accurately predicts the failure points with no misjudgment except for the second test run. The last column lists the probability of failure, P_f, calculated from the kriging model prediction. One can see from the 10 test runs, the results obtained are quite consistent with small variations. It is to be noted that for both Examples 1 and 2, all of the kriging model predictions for 10,000 MC sample points are validated by calling the expensive function. Therefore the obtained P_f values in both Examples 1 and 2 can be considered as if computed from the expensive performance function.

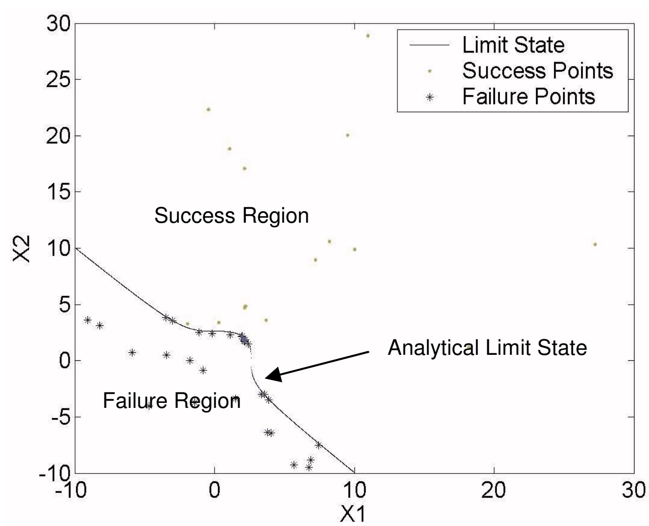

Figure 2 Distribution of the evaluated points in the design space for Example 1.

The obtained results are compared with the best results from the literature in Table 2 (Zou et al. 2002). As one can see for the same magnitude of accuracy, the proposed method only needs a very small fraction of the total number of function evaluations.

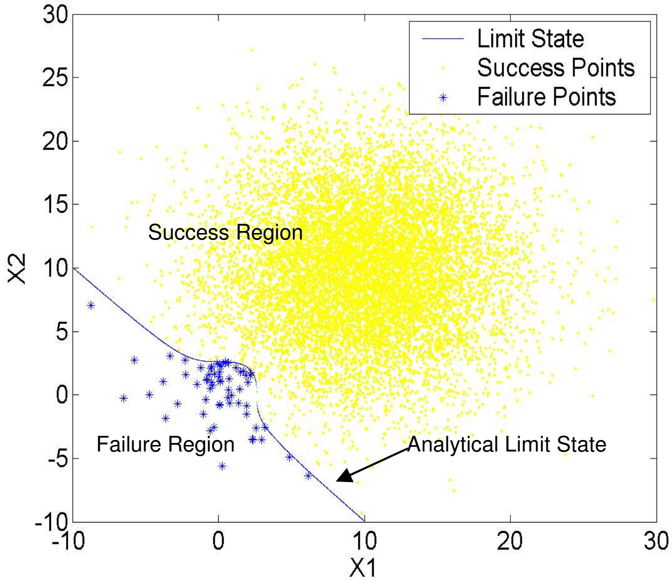

Figure 3 MCS results on the kriging model with 10,000 samples for Example 1.

All of the evaluated expensive points are plotted in Figure 2 with respect to the analytical limit state. One can see that many points are generated near the limit state in the failure region, as expected. Figure 3 illustrated the MCS points on the kriging model with respect to the analytical limit state. As one can see that all the predicted failure points are indeed in the real failure region, indicating a zero misjudgment situation.

EXAMPLE 2

Example 2 (Zou et al. 2003) is formulated from a tuned vibration absorber (TVA) system. The amplitude of the system is the performance function described as below:

$$f(\beta_1,\beta_2)=\frac{\left|1-\left(\frac{1}{\beta_2}\right)^2\right|}{\sqrt{\left[1-R\left(\frac{1}{\beta_1}\right)^2-\left(\frac{1}{\beta_1}\right)^2-\left(\frac{1}{\beta_2}\right)^2+\frac{1}{\beta_1^2\beta_2^2}\right]^2+4\varsigma^2\left[\frac{1}{\beta_1}-\frac{1}{\beta_1\beta_2^2}\right]^2}} \qquad (9)$$

Where R is the mass ratio of the absorber to the original system, ς is the damping ratio of the original system, and β_1 and β_2 are the ratios of the natural frequency of the original system and vibration absorber with respect to the excitation frequency, respectively. In this work, R and ς are treated as deterministic variables with R=0.01 and ς=0.01; only β_1 and β_2 are random variables with a distribution $\beta_1 \sim N(1, 0.025)$ and $\beta_2 \sim N(1, 0.025)$. The objective of the design problem is to reduce the risk of the amplitude being larger than a certain value, under the uncertainties of the parameters. The limit state function for this example is

$$g(\beta_1,\beta_2)=28.0-f(\beta_1,\beta_2) \qquad (10)$$

This problem involves multiple failure regions and was deemed "extremely difficult to solve with existing methods" except for the MC method (Zou et al. 2003).

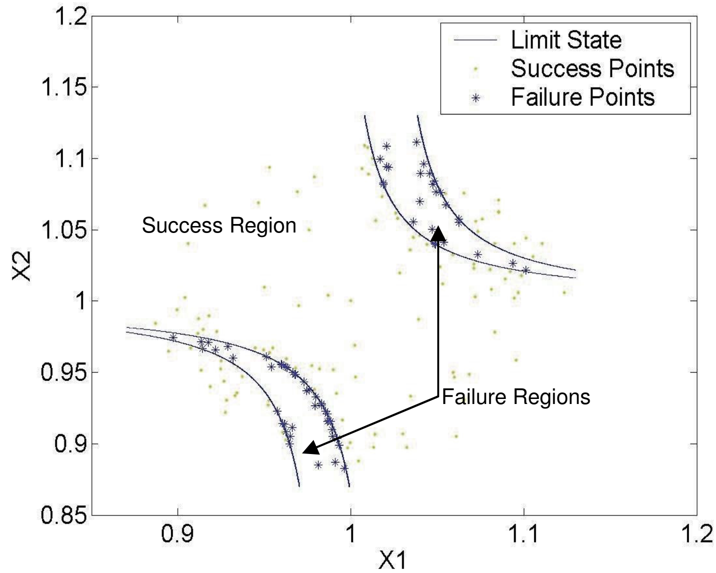

Figure 4 Distribution of the evaluated points in the design space for Example 2.

Test results are listed in Table 3 with $\varepsilon_1 = 0.1; \varepsilon_2 = 0.1$, which is organized in the same manner as Table 1. The major difference is in the 8th column. These exists two situations of misjudgment. One is that the point is in fact successful but misjudged as failure, and the other is that the point is in fact failure but misjudged as success. A negative sign is used to indicate the former situation; and a positive sign is used for the latter. For example for the 2nd test run, there are 7 misjudged points. Among these 7 points, 4 points are mistakenly judged as failure points while in fact they are in the success region, and 3 points are mistakenly judged as successful. One can see that for the 4th and 9th test runs, the number of misjudged points is large. By plotting the evaluated points, it is found that the method converges either before both failure regions are identified or before the profiles of the regions are completely captured. Roughly from the test results, there is about one fifth of the chance that the method will miss one of the failure regions. This issue raises concerns. On the other hand, the proposed method found both failure regions in 8 out of 10 runs. The median estimated P_f is very close to the result obtained by MCS, as shown in Table 4.

As shown in Table 4, the results are compared with the best results in literature. It is found that the multi-modal AIS method needs a large number of function evaluations and it missed one of the failure regions. The recent IRS-based MC method also requires about double the amount of function evaluations than the proposed method for a similar accuracy. Also both the proposed method and the IRS-based MC method have achieved a very high accuracy of less than 1% to the value obtained by MCS.

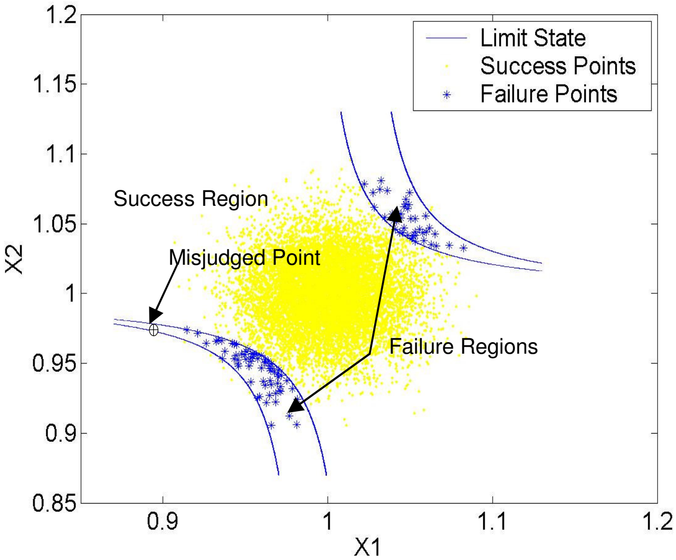

Figure 5 MCS results on the kriging model with 10,000 samples for Example 2.

Similar to Example 1, Figure 4 plots all the evaluated points. It is clear that more points are on or near the limit state. Figure 5 illustrates the MCS results on the kriging model with one outlier as shown in the figure as "misjudged point".

DISCUSSIONS

For reliability assessment of expensive performance functions, the importance sampling (IS) is probably the most efficient method thus far (Au and Beck 1999; Zou et al. 2002; Schuëller et al. 2003). There are two types of IS methods. One is the IS on design points and the other is IS on kernel sampling density (Schuëller et al. 2003).

IS on the design point is to move the center of sampling to a point on the limit state, which is usually the conventional MPP. This method was promoted by (Du and Chen 2000). The method, however, bears a few problems:

1. In general, the search for design points occupies a considerable portion of the total computational effort.
2. If there are multiple MPPs, the search for multiple design points requires more sophisticated algorithms for the optimization problem. Moreover, since it is not known a priori, multiple points search has to be carried out for every problem.
3. The application of design points to IS becomes more difficult or inefficient in situations such as noisy limit state functions, relatively flat PDFs along or in the neighborhood of the limit state surfaces, and highly concave/convex limit state surfaces. When such situations are not properly handled in IS, the estimate can have a large variance or even become practically biased (Au and Beck 1999) .

The IS on kernel sampling density is to use sample points to construct a kernel density estimator of the optimal IS density. The main drawback is that the points used to construct the kernel sampling density are simulated by the basic Monte Carlo procedure, so the probability of having samples generated in the failure region is equal to the failure probability, which is usually small in practical applications. The simulation of points lying in the failure region thus requires a very large number of samples and so the method is computationally expensive. The version of IS on kernel density in (Au and Beck 1999) was considered the most efficient approach of this category by (Schuëller et al. 2003).

Comparatively, the proposed method has a few distinctive features:

1. The sampling process is straightforward. Compared to the IS method, the proposed sampling strategy is simple and easy to understand by practitioners. Furthermore, this strategy is not restricted to any parametric family of densities as in the case of conventional IS methods.
2. The MPP is re-defined in this work as the point in the failure region that has the largest JPDF value in the original design space.
3. There is no need to perform transformation in order to calculate the MPP. Since the method works always in its original design space, no transformation and thus approximation of the variable distribution is needed. This eliminates the possible error in calculating the MPP in the transformed normal space.
4. There is no need to identify the MPP first. The proposed method identifies MPP along with other points on and close to the limit state.
5. This method only focuses on an accurate kriging model near the limit state rather than globally, therefore the cost of metamodeling building is reduced.
6. The proposed method does not call any MCS process directly on the expensive performance function.

Also it is noted that the proposed method bears certain limitations. As shown from Example 2, the proposed method may miss some of the failure regions when multiple failure regions are present. This could be remedied by having more initial sample points or tightening the convergence criteria by reducing ε_i, $i = 1, \cdots 3$ values. It could be argued that the chance of missing a failure region for the proposed method would not be higher than the IS method on design points. It is because for the proposed method, as long as there is one evaluated point in the failure region, its sampling guidance function value would be high and more sample points would be generated around it. Therefore, as long as there is one evaluated point in a failure region, more sample points will be generated immediately in the region. For global optimization methods on black-box functions, if there lacks of information in the neighborhood of a global optimum, this optimum will likely be missed regardless of which algorithm is used. Thus when using global optimization methods to search for all the MPPs, the situation of missing a MPP is qualitatively similar to missing a failure region in the proposed method. For IS, if one MPP is missed, it will miss a failure region. Such a situation happens as shown in Table 4 (Zou et al. 2003). Another challenge for the proposed method is for high dimension problems since it employs the kriging metamodel. As the number of sample points increases with dimensions, the cost of building a kriging model will increase. As the final P_f is estimated from the kriging model, the possibility of misjudgment might increase as well. The “curse of dimensionality”, however, is a challenge to almost all existing numerical methods.

CONCLUSION

This work developed a simple discriminative sampling method integrated with metamodeling in achieving a more efficient reliability assessment. Its effectiveness and efficiency are demonstrated through tests. The test results also represent the state-of-the-art for reliability assessment of expensive performance functions. The proposed method is found very accurate for problems of single failure region. It also has a high success rate for problems of multiple failure regions. Future research will develop more reliable methods for problems of multiple failure regions, as well as reliability assessment for high dimensional problems.

ACKNOWLEDGMENTS

Financial support from Canadian Natural Science and Engineering Research Council (NSERC) is gratefully appreciated.

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Table 1 Results for 10 test runs for Example 1.

Test No.	Evaluated MPP		Model MPP		# of Iter.	# of Func. Eval.	# of Mis-judgment (in 10,000 points)	P_f (x 10^{-4})
	JPDF (x 10^{-4})	X	JPDF (x 10^{-4})	X				
1	5.2270	2.0927 1.9969	5.0889	2.1309 1.8748	14	40	0	57
2	5.1576	2.2568 1.7945	5.2273	2.1401 1.9497	15	44	1	51
3	5.0334	2.0373 1.9332	5.2730	2.0919 2.0255	8	27	0	60
4	4.9937	2.1150 1.8317	5.2273	2.1401 1.9497	11	35	0	65
5	5.3247	1.9865 2.1640	5.1715	2.0145 2.0416	13	40	0	55
6	5.3260	2.1318 2.0171	5.2251	2.0683 2.0200	42	97	0	61
7	5.2885	2.0867 2.0399	5.2539	1.9563 2.1521	37	86	0	47
8	5.2885	2.0867 2.0399	5.1971	2.0425 2.0290	13	39	0	56
9	5.2274	2.0927 1.9969	4.8688	1.9733 1.8932	19	49	0	58
10	5.3247	1.9865 2.1640	4.9256	1.7980 2.1100	17	44	0	60
Median	5.2580		5.2111		14.5	42	0	57.5

Table 2 Result comparison with the best results in literature for Example 1.

$P_{f,\,MCS}$=0.00566 with 120,000 MCS samples (Zou et al. 2002)			
	P_f	$(P_f - P_{f,\,MCS})/P_{f,\,MCS}$	# of Func. Eval.
Multi-modal AIS (Zou et al. 2002)	0.00575	1.6%	680 (including 120 eval. to search the MPP)
Proposed Method	**0.00575**	**1.6%**	**42**

Table 3 Results for 10 test runs for Example 2.

Test No.	Evaluated MPP		Model MPP		# of Iter.	# of Func. Eval.	# of Mis-judgment (in 10,000 points)	P_f
	JPDF	X	JPDF	X				
1	14.8448	0.9630 0.9532	14.8516	0.9624 0.9537	89	192	0	0.0108
2	14.8464	0.9630 0.9533	14.8334	0.9627 0.9535	49	113	7 (-4 +3)	0.0093
3	14.8255	0.9625 0.9537	14.7928	0.9638 0.9526	87	193	4 (-2 +2)	0.0104
4	14.7554	0.9637 0.9527	14.6623	0.9630 0.9531	36	87	51 (-2 +49)	0.0053 (missed)
5	14.8469	0.9622 0.9539	14.8157	0.9619 0.9541	192	409	13 (-1 +12)	0.0126
6	14.8316	0.9630 0.9533	14.7700	0.9617 0.9547	52	119	17 (-5 +12)	0.0098
7	14.7984	0.9637 0.9527	14.7435	0.9645 0.9521	34	81	10 (-5 +5)	0.0096
8	14.7449	0.9617 0.9542	14.7581	0.9635 0.9528	17	51	25 (-2 +23)	0.0103
9	14.8281	0.9624 0.9537	14.7286	0.9613 0.9545	19	49	44 (-2 +42)	0.0073 (missed)
10	14.8250	0.9629 0.9533	14.7840	0.9638 0.9526	40	96	11 (-5 +6)	0.0097
Median	14.8286		14.7770		45	105	12	0.00975

Table 4 Result comparison with the best results in literature for Example 2.

$P_{f,\,MCS}$=0.0097 with 30,000 MCS samples (Zou et al. 2003)			
	P_f	$(P_f - P_{f,\,MCS})/P_{f,\,MCS}$	# of Func. Eval.
Multi-modal AIS (Zou et al. 2003)	0.00407	58%	840
IRS-Based MC (Zou et al. 2003)	0.00963	0.72%	206
Proposed Method	**0.00975**	**0.52%**	**105**