CENSORED LINEAR REGRESSION MODELS

Censored linear regression models constitute a class of special limited dependent variable* models where the dependent variables are censored. The censoring* can be from above or below; the censoring points can be known, unknown, or random. For illustration, consider a model where the dependent variable is censored from below at the known point zero; precisely,

$$\begin{aligned} \eta_t &= \beta_1 + \mathbf{x}_t'\boldsymbol{\beta}_2 + u_t, \\ y_t &= \max\{\eta_t, 0\}, \end{aligned} \tag{1}$$

where $\eta_t \in \mathbb{R}$ is the censored dependent variable, y_t is its observed counterpart, $\mathbf{x}_t \in \mathbb{R}^k$ is the observed vector of covariates, $\boldsymbol{\beta} = (\beta_1, \boldsymbol{\beta}_2')'$ the vector of regression parameters, and u_t the errors in the regression equation. It is usually assumed that u_t is normally distributed with mean 0 and variance σ_u.

This model was first applied in economics by Tobin [22] and is known as the *Tobit model* in the econometric literature [10]; *see* PROBIT ANALYSIS. For the "standard" Tobit model (1), many statistical inference methods have been developed in the last twenty years References may be found, e.g., in refs. 2, 3, 14, 18. The related models are Probit models and truncated linear regression models.

It is well known that the ordinary least squares (OLS) estimators of $\boldsymbol{\beta}$ in the model (1), using either all observations or only the positive observations of y_t and the corresponding observations of $\mathbf{x}_t$, are biased and inconsistent [3,11]. Goldberger [11], Greene [14], and Chung and Goldberger [6] proposed corrected OLS estimators of $\boldsymbol{\beta}$ under the extra assumption that $\mathbf{x}_t$ is normally distributed. Heckman [15] proposed a two-stage estimator of a more general model of (1). Amemiya [1] derived the asymptotic normality* of the maximum likelihood estimators* (MLEs) of $\boldsymbol{\beta}$ and σ_u. Using Tobin's reparameterization, Olsen [19] showed that a reparametrized likelihood function of the model (1) has a unique, global, finite maximum. Powell [20,21] proposed a least absolute deviation* (LAD) estimator of the model (1) and derived its consistency and asymptotic distribution. A Bayes approach was used by Chib [5].

TOBIT MODEL WITH MEASUREMENT ERRORS

In many practical applications the explanatory variable $\mathbf{x}_t$ or some of its components are not or cannot be exactly measured. In such cases a more general model than (1) is desirable. A censored linear errors-in-variables* (CEV) model can be defined as

$$\begin{aligned} \eta_t &= \beta_1 + \boldsymbol{\beta}_2'\boldsymbol{\xi}_t + u_t, \\ y_t &= \max\{\eta_t, 0\}, \\ \mathbf{x}_t &= \boldsymbol{\xi}_t + \mathbf{v}_t, \end{aligned} \tag{2}$$

where instead of $\boldsymbol{\xi}_t, \mathbf{x}_t$ is actually observed, and $\mathbf{v}_t$ represents measurement errors*. Furthermore, $u_t, \mathbf{v}_t$ and $\boldsymbol{\xi}_t$ are assumed to be independently and normally distributed with means 0, $\mathbf{0}, \boldsymbol{\mu}\xi$ and variances $\sigma_u, \boldsymbol{\Sigma}_v, \boldsymbol{\Sigma}_\xi$ respectively. If the measurement error covariance $\boldsymbol{\Sigma}_v = \mathbf{0}$, then the model (2) reduces to the error-free model (1).

The problem with measurement errors has received more attention recently. Weiss [26] considered the LAD estimator of model (2). Wang [23,24,24] investigated the MLE and proposed two-step moment estimators of the model. A closely related binary choice model with measurement errors has been studied [16], and also a class of more general stochastic frontier models [7].

IDENTIFIABILITY

The usual linear normal errors-in-variables model is not identifiable, and hence the model cannot be estimated consistently. The model (2) is nonlinear because of censoring. However, this censoring can only help to identify the lost information caused by censoring itself and does not help to identify all model parameters [24]. Specifically, in the model (2) only the parameters $(\mu_\eta, \sigma_\eta, \boldsymbol{\sigma}_{x\eta}, \boldsymbol{\mu}_\xi, \boldsymbol{\mu}_x, \boldsymbol{\Sigma}_x)$ are uniquely identified, whereas the parameters $(\beta_1, \boldsymbol{\beta}_2, \sigma_u, \boldsymbol{\Sigma}_\xi, \boldsymbol{\Sigma}_v)$ are not uniquely identified.

In practical applications the *a priori* identifying information is usually provided in terms of at least $k(k+1)/2$ linear restrictions on the parameters, e.g., that the variance ratio $\sigma_u^{-1}\boldsymbol{\Sigma}_v$ or the so-called reliability ratio $\boldsymbol{\kappa} = \boldsymbol{\Sigma}_x^{-1}\boldsymbol{\Sigma}_\xi$ is known or may be estimated previously. The latter is equivalent to the condition that the noise-to-signal ratio $\boldsymbol{\Delta} = \boldsymbol{\Sigma}_\xi^{-1}\boldsymbol{\Sigma}_v$ is known. This information can be obtained in many situations when, e.g., validation data, panel data or repeated sampling are available. For more discussion see [4,8,9,17]. In the following we assume that $\boldsymbol{\Delta} = \boldsymbol{\Sigma}_\xi^{-1}\boldsymbol{\Sigma}_v$ is known.

MODEL REDUCTION AND ESTIMATION

The model (2) can be reduced to an error-free form. Indeed, it can be transformed into the familiar form of a censored regression model [25]:

$$\begin{aligned} \eta_t &= \gamma_1 + \mathbf{x}_t'\boldsymbol{\gamma}_2 + w_t, \\ y_t &= \max\{\eta_t, 0\}, \end{aligned} \tag{3}$$

where w_t has distribution $N(0, \sigma_w)$ and is independent of $\mathbf{x}_t$. The relations between the new parameters $(\gamma_1, \gamma_2, \sigma_w, \boldsymbol{\mu}_x, \boldsymbol{\Sigma}_x)$ and the original ones $(\beta_1, \boldsymbol{\beta}_2, \sigma_u, \boldsymbol{\mu}_\xi, \boldsymbol{\Sigma}_\xi, \boldsymbol{\Sigma}_v)$ are given by

$$\begin{aligned} &\beta_1 = \gamma_1 - \boldsymbol{\mu}_x'\boldsymbol{\Delta}\boldsymbol{\gamma}_2, \qquad \boldsymbol{\beta}_2 = (\mathbf{I} + \boldsymbol{\Delta})\gamma_2, \\ &\sigma_u = \sigma_w - \boldsymbol{\gamma}_2'\boldsymbol{\Sigma}_x\boldsymbol{\Delta}\boldsymbol{\gamma}_2, \qquad \boldsymbol{\mu}_\xi = \boldsymbol{\mu}_x, \\ &\boldsymbol{\Sigma}_\xi = \boldsymbol{\Sigma}_x(\mathbf{I} + \boldsymbol{\Delta})^{-1}. \end{aligned} \tag{4}$$

The mapping (4) is one-to-one; consequently, any estimator of the model (3) implies a corresponding estimator of the model (2). Using this approach, it is also possible to derive the asymptotic bias of the estimator of (2) when the identifying information $\boldsymbol{\Delta}$ is misspecified. Suppose, for instance that $\hat{\boldsymbol{\psi}} = (\hat{\gamma}_1, \hat{\boldsymbol{\gamma}}_2', \hat{\sigma}_w)'$ is a consistent estimator of (3) and $\tilde{\boldsymbol{\theta}} = (\tilde{\beta}_1, \tilde{\boldsymbol{\beta}}_2', \tilde{\sigma}_u)'$ is obtained via (4) where, instead of $\boldsymbol{\Delta}$, a wrong $\tilde{\boldsymbol{\Delta}}$ is used. Then, the asymptotic bias of $\tilde{\boldsymbol{\theta}}$ is given by

$$\begin{aligned} \operatorname{plim} \tilde{\beta}_1 &= \beta_1 + \boldsymbol{\mu}_x'(\boldsymbol{\Delta} - \tilde{\boldsymbol{\Delta}})(\mathbf{I} + \boldsymbol{\Delta})^{-1}\boldsymbol{\beta}_2, \\ \operatorname{plim} \tilde{\boldsymbol{\beta}}_2 &= \boldsymbol{\beta}_2 - (\boldsymbol{\Delta} - \tilde{\boldsymbol{\Delta}})(\mathbf{I} + \boldsymbol{\Delta})^{-1}\boldsymbol{\beta}_2, \\ \operatorname{plim} \tilde{\sigma}_u &= \sigma_u + \boldsymbol{\beta}_2'\boldsymbol{\Sigma}_\xi(\boldsymbol{\Delta} - \tilde{\boldsymbol{\Delta}})(\mathbf{I} + \boldsymbol{\Delta})^{-1}\boldsymbol{\beta}_2. \end{aligned}$$

Consequently, the estimation biases are of the same order as $\boldsymbol{\Delta} - \tilde{\boldsymbol{\Delta}}$ and hence can be significant if the amount of misspecification $\boldsymbol{\Delta} - \tilde{\boldsymbol{\Delta}}$ is not very small relative to $\mathbf{I} + \boldsymbol{\Delta}$. Further, the slope parameter $\boldsymbol{\beta}_2$ tends to be underestimated by underspecified $\boldsymbol{\Delta}$ and overestimated by overspecified $\boldsymbol{\Delta}$, whereas the converse is true for β_1 and σ_u.

The model (3) is different from the ordinary Tobit model (1) in that the $\mathbf{x}_t$ in (3) is a random vector and is unbounded under normality, whereas in the Tobit model it is usually assumed to consist of bounded constants. This fact should be taken into account in deriving the asymptotic covariance matrices of the estimators, e.g., of the maximum likelihood estimator.

A TWO-STEP MOMENT ESTIMATOR (TME)

Suppose the data $(y_t, \mathbf{x}_t'), t = 1, 2, \ldots, T,$ are i.i.d. Denote the sample moments by

$$\hat{\mu}_y = \frac{1}{T}\sum_{t=1}^{T} y_t, \qquad \hat{\boldsymbol{\mu}}_x = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_t,$$

$$\hat{\boldsymbol{\Sigma}}_x = \frac{1}{T}\sum_{t=1}^{T}(\mathbf{x}_t - \overline{\mathbf{x}})(\mathbf{x}_t - \overline{\mathbf{x}})'.$$

Let $\mu_{y+} = E(y_t|y_t > 0)$ and $\boldsymbol{\mu}_{xy+} = E(\mathbf{x}_t y_t|y_t > 0)$. These conditional moments are consistently estimated by the corresponding sample moments, using the positive y_t's and the corresponding $\mathbf{x}_t$'s. These estimators are denoted analogously as $\hat{\mu}_{y+}, \hat{\boldsymbol{\mu}}_{xy+}$, respectively. Finally, let $\delta = \mu_\eta/\sqrt{\sigma_\eta}$, and let $\Phi(\cdot)$ and $\phi(\cdot)$ denote the standard normal distribution and density functions.

Wang [25] proposed a two-step estimation procedure. In the first step, the first and second moments of $(\eta_t, \mathbf{x}_t')$ are estimated by

$$\begin{aligned} \hat{\mu}_\eta &= \frac{\hat{\delta}\hat{\mu}_{y+}}{\hat{\delta} + \phi(\hat{\delta})/\Phi(\hat{\delta})}, \\ \hat{\sigma}_\eta &= (\hat{\mu}_\eta/\hat{\delta})^2, \\ \hat{\boldsymbol{\sigma}}_{x\eta} &= \hat{\boldsymbol{\mu}}_{xy+} - \hat{\boldsymbol{\mu}}_x\hat{\mu}_{y+}, \end{aligned}$$

where $\hat{\delta} = \Phi^{-1}(\hat{\mu}_y/\hat{\mu}_{y+})$. In the second step, the remaining parameters are estimated by

$$\hat{\boldsymbol{\beta}}_2 = \hat{\boldsymbol{\Sigma}}_\xi^{-1}\hat{\boldsymbol{\sigma}}_{x\eta}, \qquad \hat{\beta}_1 = \hat{\mu}_\eta - \hat{\boldsymbol{\beta}}_2'\hat{\boldsymbol{\mu}}_\xi,$$
$$\hat{\sigma}_u = \hat{\sigma}_\eta - \hat{\boldsymbol{\beta}}_2'\hat{\boldsymbol{\sigma}}_{x\eta}, \qquad \hat{\boldsymbol{\mu}}_\xi = \hat{\boldsymbol{\mu}}_x,$$
$$\hat{\boldsymbol{\Sigma}}_\xi = \hat{\boldsymbol{\Sigma}}_x(\mathbf{I} + \boldsymbol{\Delta})^{-1}.$$

All these estimators are strongly consistent, because they are continuous functions of the sample moments. Furthermore, they are asymptotically normally distributed [25]. The asymptotic covariance matrices given in ref. 25 apply to the moment estimators of the Tobit model (1) as well, because it is a special case of the model (2) with $\boldsymbol{\Delta} = \mathbf{0}$ (and hence $\boldsymbol{\Sigma}_\xi = \boldsymbol{\Sigma}_x$).

This two-step procedure may be similarly applied to the case where, instead of $\boldsymbol{\Delta}$, the variance ratio $\sigma_u^{-1}\boldsymbol{\Sigma}_v$ is known. The only difference is that the second-step estimators should be calculated similarly as in ref. 8. The asymptotic results of the estimators may be established analogously. Such results for a simple model with $k = 1$ are given by Theorem 1.3.1 of ref. 8.

MAXIMUM LIKELIHOOD ESTIMATOR (MLE)

Without loss of generality, let the data be given as in the preceding section, in which the first $T_0 y_t$'s are zero, and the last $T_1 = T - T_0\, y_t$'s are positive. The MLE of $\boldsymbol{\mu}_x$ and $\boldsymbol{\Sigma}_x$ are given by the corresponding sample moments, and the MLE of $\boldsymbol{\mu}_\xi$ and $\boldsymbol{\Sigma}_\xi$ are therefore identical with the TME [24]; analogously to ref. 19, the reparametrized conditional log-likelihood function

$$L_c(\boldsymbol{\psi}) = \sum_{t=1}^{T_0} \log \Phi(-\boldsymbol{\alpha}'\tilde{\mathbf{x}}_t) + T_1 \log \tau - \tfrac{1}{2}\boldsymbol{\psi}'\mathbf{Z}'\mathbf{Z}\boldsymbol{\psi} \qquad (5)$$

is globally concave in $\boldsymbol{\psi} = (\boldsymbol{\alpha}', \boldsymbol{\tau})' \in \mathbb{R}^{k+1} \times \mathbb{R}_+$, where $\mathbb{R}_+ = (0, +\infty), \tilde{\mathbf{x}}_t = (1, \mathbf{x}_t')', \mathbf{Z} = (\mathbf{X}_1, -\mathbf{Y}_1), \mathbf{X}_1 = (\tilde{\mathbf{x}}_{T_0+1}, \tilde{\mathbf{x}}_{T_0+2}, \ldots, \tilde{\mathbf{x}}_T)'$, and $\mathbf{Y}_1 = (y_{T_0+1}, y_{T_0+2}, \ldots, y_T)'$. Analogously to [1], the MLE for $\boldsymbol{\psi}$ is asymptotically normal with an asymptotic covariance matrix which is the inverse of

$$-\operatorname*{plim}_{T\to\infty} \frac{1}{T}\frac{\partial^2 L_c(\boldsymbol{\psi})}{\partial\boldsymbol{\psi}\,\partial\boldsymbol{\psi}'} = \boldsymbol{\Omega} = \boldsymbol{\Omega}_0 + \Phi(\delta)\boldsymbol{\Omega}_1,$$

where $\delta = \mu_\eta/\sqrt{\sigma_\eta}$,

$$\boldsymbol{\Omega}_0 = \begin{pmatrix} \Phi(-\delta)E[\lambda_t(\lambda_t - \boldsymbol{\alpha}'\tilde{\mathbf{x}}_t) & 0 \\ \times\tilde{\mathbf{x}}_t\tilde{\mathbf{x}}_t'|\eta_t \leqslant 0] & \\ 0 & \Phi(\delta)/\tau^2 \end{pmatrix},$$

$$\boldsymbol{\Omega}_1 = \begin{pmatrix} E(\tilde{\mathbf{x}}_t\tilde{\mathbf{x}}_t'|y_t > 0) & -E(\tilde{\mathbf{x}}_t y_t|y_t > 0) \\ -E(y_t\tilde{\mathbf{x}}_t'|y_t > 0) & E(y_t^2|y_t > 0) \end{pmatrix},$$

and $\lambda_t = \phi(\boldsymbol{\alpha}'\tilde{\mathbf{x}}_t)/\Phi(-\boldsymbol{\alpha}'\tilde{\mathbf{x}}_t)$ [24]. The MLE for $\boldsymbol{\theta} = (\beta_1, \boldsymbol{\beta}_2', \sigma_u)'$ is calculated according to

$$\beta_1 = (\alpha_1 - \boldsymbol{\mu}_x'\boldsymbol{\Delta}\boldsymbol{\alpha}_2)/\tau,$$
$$\boldsymbol{\beta}_2 = (\mathbf{I} + \boldsymbol{\Delta})\boldsymbol{\alpha}_2/\tau,$$
$$\sigma_u = (1 - \boldsymbol{\alpha}_2'\boldsymbol{\Sigma}_x\boldsymbol{\Delta}\boldsymbol{\alpha}_2)/\tau^2.$$

It is clear that $\hat{\boldsymbol{\theta}}_{\mathrm{ML}}$ is consistent. Further, $\sqrt{T}(\hat{\boldsymbol{\theta}}_{\mathrm{ML}} - \boldsymbol{\theta}_0) \xrightarrow{L} N(0, \sigma_w\mathbf{C}\boldsymbol{\Omega}^{-1}\mathbf{C}')$, where $\sigma_w = \sigma_u + \boldsymbol{\beta}_2'\boldsymbol{\Sigma}_v(\mathbf{I} + \boldsymbol{\Delta})^{-1}\boldsymbol{\beta}_2$ and

$$\mathbf{C} = \begin{pmatrix} 1 & -\boldsymbol{\mu}_x'\boldsymbol{\Delta} & -\beta_1 \\ 0 & \mathbf{I} + \boldsymbol{\Delta} & -\boldsymbol{\beta}_2 \\ 0 & -2\boldsymbol{\beta}_2'\boldsymbol{\Sigma}_x\boldsymbol{\Delta} & -2\sigma_u \end{pmatrix}.$$

The maximization of $L_c(\boldsymbol{\psi})$ may be carried out through standard numerical methods such as Newton-Raphson*. The numerical calculation is straightforward, as the first and second derivatives of $L_c(\boldsymbol{\psi})$ are available:

$$\frac{\partial L_c(\boldsymbol{\psi})}{\partial\boldsymbol{\psi}} = \begin{pmatrix} -\mathbf{X}_0'\boldsymbol{\lambda}_0 \\ T_1/\tau \end{pmatrix} - \mathbf{Z}'\mathbf{Z}\boldsymbol{\psi},$$

$$\frac{\partial^2 L_c(\boldsymbol{\psi})}{\partial\boldsymbol{\psi}\,\partial\boldsymbol{\psi}'} = -\begin{pmatrix} \mathbf{X}_0'\boldsymbol{\Lambda}_0\mathbf{X}_0 & \mathbf{0} \\ \mathbf{0} & T_1/\tau^2 \end{pmatrix} - \mathbf{Z}'\mathbf{Z},$$

where $\mathbf{X}_0 = (\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \ldots, \tilde{\mathbf{x}}_{T_0})', \boldsymbol{\lambda}_0 = (\lambda_t, t = 1, 2, \ldots, T_0)', \lambda_t = \phi(\boldsymbol{\alpha}'\tilde{\mathbf{x}}_t)/\Phi(-\boldsymbol{\alpha}'\tilde{\mathbf{x}}_t)$, and $\boldsymbol{\Lambda}_0$ is the diagonal matrix with diagonal elements $\lambda_t(\lambda_t - \boldsymbol{\alpha}'\tilde{\mathbf{x}}_t), t = 1, 2, \ldots, T_0$.

Since $L_c(\boldsymbol{\psi})$ is globally concave, the iteration may start at any finite point. However, a good starting point is important for rapid convergence; the TME of the preceding section may serve as initial values for the iterations. As is shown by a Monte Carlo study in ref. 24, for a simple model with $k = 1$ the MLE procedures using the Newton-Raphson algorithm and the TME as starting values may achieve rather satisfactory convergence after four or five iterations. The estimators $\hat{\boldsymbol{\psi}}_1$ and $\hat{\boldsymbol{\theta}}_1$ obtained after one iteration of the Newton-Raphson procedure have the same asymptotic distributions as the MLEs $\hat{\boldsymbol{\psi}}_{\mathrm{ML}}$ and $\hat{\boldsymbol{\theta}}_{\mathrm{ML}}$, respectively.

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See also Censoring; Errors in Variables; Limited Dependent Variables Models; Measurement Error; and Probit Analysis.

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