# Technical Appendix: International Risk Sharing in Emerging Economies

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## 1 Shocks to Trend Growth and Risk Sharing

In this section we examine the implications of shocks to trend growth on international business cycles and risk sharing using two approaches. First, we use impulse response analysis taking as a baseline a symmetric version of the two-country model developed in the previous sections. Specifically, the baseline model assumes elastic trade and uses an identical transitory productivity process for both economies as in country ($A$). We then add a shock to trend in country ($E$) where its volatility has the same magnitude as that of the transitory shock. Second, we use the baseline model extended with a shock to trend growth to test the sensitivity of key international moments to the relative importance of the volatility of trend-to-transitory shocks. Furthermore, we use an additional metric to assess the distance from perfect risk sharing, which we borrow from Viani 2010, who defines a measure of the efficiency gap between countries as the difference in the shadow price of income between countries (Gap). In essence, under complete financial markets the gap is always zero and there is perfect risk sharing. However, when markets are incomplete the gap may deviate from zero and risk sharing is then imperfect.

The IRFs of the baseline specification are shown in Figures 2A and 2B, which illustrate the responses of relative quantities and prices to a positive (one standard deviation) transitory TFP shock to (home) country (E) output. The first row shows the standard results from an increase in home output. Consumption increases less than output (C/Y (E)), investment increases more than output (I/Y (E)), and there is a real exchange rate depreciation that results in a surplus of the trade balance (TB/Y (E)). The domestic depreciation also helps increase country (A)'s consumption and investment relative to income. There is a trade balance deficit in country (A) (TB/Y (A)) that counters the trade surplus in country (E).

The last row shows that domestic relative consumption increases (C(E)/C(A)) and the difference between the shadow price of income across countries (Gap (E)) is positive on impact. The latter response implies that the substitution effect due to the real exchange rate depreciation (RER (E)) also results in an increase in consumption in country (A), which indicates that the risk sharing channel is positive.

Last, panel 2B shows the responses of labor market variables. A positive productivity shock in country (E) increases the real wage (W (E)) as well as labor (L (E)). In contrast, in country (A), the rise of the real wage is due to the increase in the demand, as agents in country (A) become relatively wealthier, their labor supply drops (L (A)).

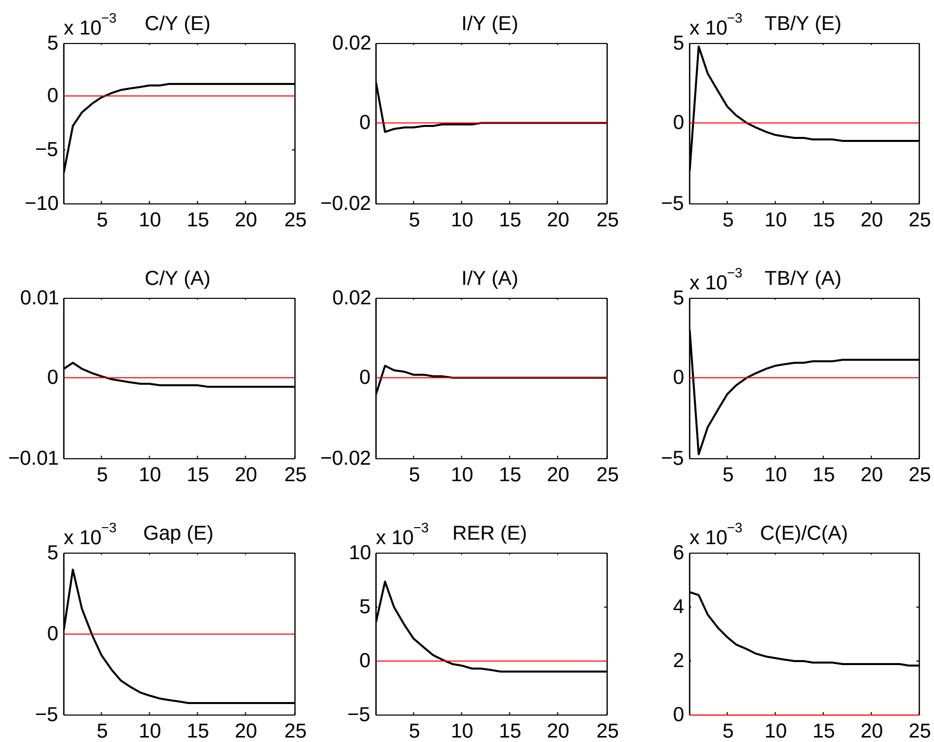


Figure 2A. Dynamic responses to transitory productivity shock in country E

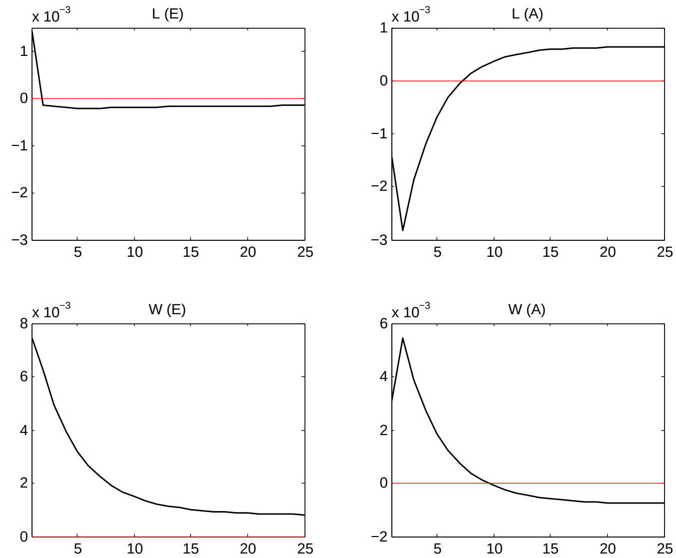


Figure 2B. Labor market response to transitory productivity shock in country E

Next, Figure 3A illustrates the responses to a positive shock to *trend growth in country (E)*. Notice that the IRFs show markedly distinct responses from those of a transitory shock. Notably, in terms of international price movements, there is a real exchange rate appreciation on impact when home agents expect income to increase permanently. What drives this result? As firms expect a higher rate of growth of output, they sharply increase their investment spending (I/Y (E)) pushing prices up. To finance their investment, firms in country (E) borrow in the international market and the trade balance becomes negative. Hence, the increase of domestic prices leads to a real exchange rate appreciation (RER (E)) along with a trade deficit (TB/Y (E)), which is matched by a trade surplus in country (A) (TB/Y (A)). Furthermore, the Gap (E) is negative on impact and relative consumption increases (C(E)/C(A)), which indicates that domestic households are relatively wealthier. The expectation of higher output growth also drives up domestic consumption, but not as much as home output (C/Y (E)). Notably, since the increase in home wealth is enhanced by a rise in prices of the domestic good, the risk sharing channel is negative. On the other hand, in country (A) the real exchange rate appreciation results in a negative wealth effect as consumption and investment fall relative to income.

Last, we examine the impulse responses of labor market variables in Figure 3B. In response to a positive shock to trend growth, country (E)'s real wage goes up and labor increases (L (E)). In contrast, in country (A), the drop in consumption and investment lowers the real wage (W (A)) and (poorer) households increase their labor supply to fulfill their trade surplus.

To summarize, in contrast to transitory shocks, the dynamic responses to a shock to trend growth have two important implications. First, there is a sharp investment response due to higher expected domestic output growth. The adjustment of international prices results in a negative wealth effect that makes agents in the domestic country relatively better off. Second, real exchange movements are sharply countercyclical. Both of these features consistent with the documented evidence on lower risk sharing in emerging economies.

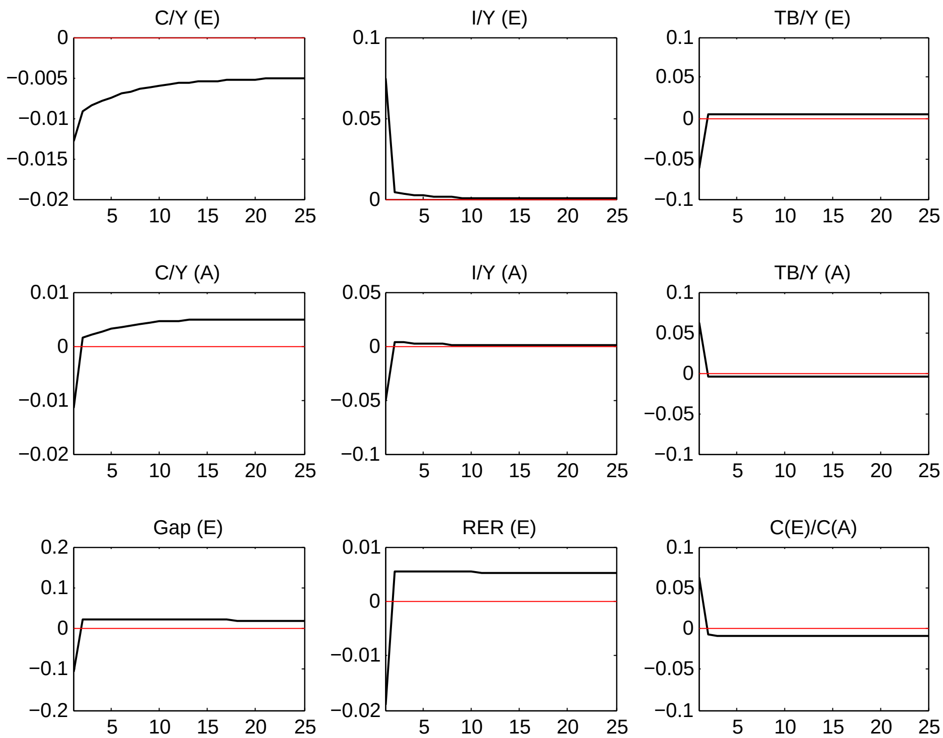


Figure 3A. Dynamic responses to trend productivity growth shock in country (E)

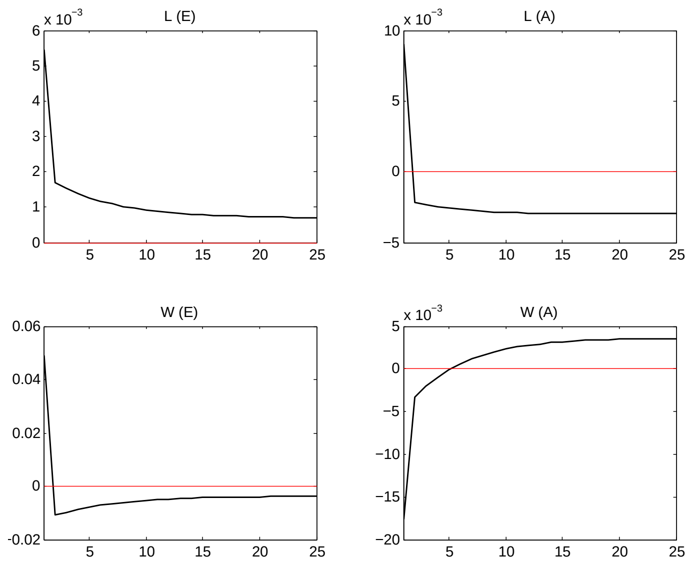


Figure 3B. Labor market response to trend productivity growth shock in country (E)

# 2 Robustness

What explains the lower observed measures of risk sharing in EMEs? To answer this question we examine key mechanisms that impinge on international risk sharing under two different market structures: Complete markets (Arrow-Debreu, $AD$) and incomplete markets (Bond Economy, $BE$). Table 4, columns 2 to 8 show seven specifications of the model categorized according to a combination of market structure and transmission mechanism, in which two baseline specifications are based on transitory shocks only ($AD(1)$ and $BE(4)$). The different sub-categories are: Elastic trade $\omega = 1.5$, Inelastic trade $\omega = 0.6$, No-Trend shocks ($NTS$), and Trend shocks ($TS$). In addition, we examine the impact of asymmetric trade shares when country $A$ is less open than country $E$ ($a_{h,A} > a_{h,E}$).

Notice several salient features. First, as expected, all the $AD$ specifications imply perfect risk sharing as measured by the Backus-Smith statistic. Second, under $AD$, *trend shocks* lead to i) a doubling of relative consumption volatility in the emerging economy; ii) large trade balance ratio volatilities in both countries, iii) a strongly pro-cyclical real exchange rate volatility (counter-factual); iv) a strongly pro-cyclical trade balance in country $E$ (counter-factual); and v) a large increase in relative investment volatility in country $A$ (factual). However, as we discuss below, the impact of trend shocks becomes more relevant as we switch from complete to incomplete markets structures.

As indicated by our theory (see Appendix C), under incomplete markets, elastic trade ($BE(4)$) is not consistent with the Backus-Smith anomaly nor the quantity puzzle. However, imposing inelastic trade in specification ($BE(5)$) leads to i) a negative Backus-Smith statistic; ii) a strongly counter-cyclical real exchange rate; ii) a counter-factually higher relative volatility of consumption in country $A$ than in country $E$; iii) a drop in the relative volatilities of investment in both countries (counter-factual); and iv) a large drop in the volatility of the trade balance ratio in country $E$ (counter-factual).

Next, we examine the model specification with elastic trade and asymmetric trade share ($BE(6)$). Here we assume that country $E$ is more open to trade than country $A$ ($a_{h,A} = 0.80 > a_{h,E} = 0.75$). Notably, relative to specification ($BE(4)$), the effect of the trade share asymmetry is a large drop in the co-movement of the real exchange rate with output, a drop in the international correlations of consumption and investment, a more counter-cyclical and more volatile trade balance ratio in country $E$, closer to the data. Furthermore, relative real exchange rate volatility in country $E$ increases.

Last, we consider the specification with trend shocks and elastic trade ($BE(7)$). Relative to the specification with inelastic trade ($BE(5)$), several implications are noteworthy. First, while both inelastic trade and trend shocks are associated with a negative Backus-Smith statistic, the latter yields a strongly negative statistic ($-0.7$) than the former mechanism ($-0.3$). Second, trend shocks lead to stronger negative international-correlations of output, consumption, and investment, consistent with the data. Third, trend shocks solve the quantity puzzle while inelastic trade does not. Last, trend shocks lead to a strongly counter-cyclical trade balance ratios in both countries $E$, while inelastic trade implies a counter-factually pro-cyclical trade balance ratio in country $A$. In sum, inelastic trade, trade share asymmetries, and trend shocks tend to move the model closer to the data in terms of lower risk sharing. More importantly however, when we extend the selection of consumption risk-sharing statistics and international co-movements, our analysis suggests that shocks to trend growth provide a more compelling explanation to international business cycles in emerging market economies.

| | | | **Arrow-Debreu (AD)** | | | **Bond Economy (BE)** | | |
|---|---|---|---|---|---|---|---|---|
| *Moment* | *Data* | $AD(1)$ | $AD(2)$ | $AD(3)$ | $BE(4)$ | $BE(5)$ | $BE(6)$ | $BE(7)$ |
| *Standard deviations* | | | | | | | | |
| $\sigma_{c_{MEX}}/\sigma_{y_{MEX}}$ | 1.4 | 0.3 | 0.5 | 0.7 | 0.4 | 0.4 | 0.3 | 0.6 |
| $\sigma_{c_{US}}/\sigma_{y_{US}}$ | 0.9 | 0.4 | 0.5 | 0.4 | 0.4 | 0.7 | 0.4 | 0.8 |
| $\sigma_{i_{MEX}}/\sigma_{y_{MEX}}$ | 3.7 | 0.9 | 1.0 | 0.7 | 1.0 | 0.6 | 1.3 | 1.6 |
| $\sigma_{i_{US}}/\sigma_{y_{US}}$ | 2.8 | 0.7 | 0.4 | 2.1 | 0.8 | 0.5 | 0.7 | 1.1 |
| $\sigma_{TBY_{MEX}}$ | 2.2 | 1.7 | 2.8 | 7.1 | 1.4 | 0.3 | 1.7 | 5.6 |
| $\sigma_{TBY_{US}}$ | 0.7 | 1.7 | 2.8 | 7.1 | 1.4 | 0.3 | 1.5 | 5.0 |
| $\sigma_{RER_{MEX}}/\sigma_{y_{MEX}}$ | 4.4 | 1.1 | 2.0 | 2.2 | 1.8 | 3.6 | 2.5 | 2.7 |
| *International correlations* | | | | | | | | |
| $\rho(RER_{MEX}, \frac{C_{MEX}}{C_{US}})$ | -0.6 | 1.0 | 1.0 | 1.0 | 0.8 | -0.3 | 0.8 | -0.7 |
| $\rho(RER_{MEX}, Y_{MEX})$ | -0.6 | 0.7 | 0.5 | 0.9 | 0.4 | -0.9 | 0.1 | -0.4 |
| $\rho(Y_{MEX}, Y_{US})$ | 0.1 | 0.1 | 0.9 | 0.9 | 0.1 | 0.9 | 0.1 | -0.1 |
| $\rho(C_{MEX}, C_{US})$ | -0.1 | 0.3 | 0.2 | 0.5 | 0.5 | 1.0 | 0.4 | -0.2 |
| $\rho(I_{MEX}, I_{US})$ | 0.0 | -0.3 | 0.2 | -0.9 | -0.5 | 0.6 | -0.6 | -0.9 |
| $\rho(TBY_{MEX}, Y_{MEX})$ | -0.5 | -0.4 | -0.8 | 0.9 | -0.2 | 0.7 | -0.4 | -0.7 |
| $\rho(TBY_{US}, Y_{US})$ | -0.6 | 0.0 | 0.5 | -0.8 | 0.2 | -0.5 | 0.4 | -0.4 |

**Table 4. Mechanisms of International Risk Sharing.**
(1) $\omega = 1.5$, no trend shocks ($NTS$); (2) $\omega = 0.6$, $NTS$; (3) $\omega = 1.5$, trend shocks ($TS$);
(4) $\omega = 1.5$, $NTS$; (5) $\omega = 0.6$, $NTS$; (6) $\omega = 1.5$, asymmetric, $NTS$; (7) $\omega = 1.5$, $TS$.

Statistics are measured from the cyclical component of the HP-filtered series at annual frequency.

# 3 Risk sharing mechanisms under the lens of a basic model

In this section I analyze two well-known mechanisms that influence international risk sharing using a basic framework of an endowment two-country, two-good model under financial autarky. A novel part of the analysis is that I relax the assumption of symmetry between countries and use a measure of efficiency proposed by Viani 2010 in order to assess the impact of the mechanisms on risk sharing.

**Trade Elasticity**. Consider a two-country, two-good endowment economy under financial autarky (FA) where country A produces good 'a' and country E produces good 'b'.
In the decentralized economy country $E$ solves the following problem:[1]

$$\max_{\{C^E\}} \quad \frac{(C^E)^{1-\sigma}}{1-\sigma} + \lambda^E\{P_b^E Y_b^E - P^E C^E\}$$
$$\text{s.t.} \quad C^E = \left[(a_b^E)^{\frac{1}{\omega}} c_b^{E\,\frac{\omega-1}{\omega}} + (1-a_b^E)^{\frac{1}{\omega}} c_a^{E\,\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}$$

with the corresponding price index $P^E = \left[a_b^E p_b^{E\,1-\omega} + (1-a_b^E) p_a^{E\,1-\omega}\right]^{\frac{1}{1-\omega}}$.

The corresponding demand functions associated with the household's cost minimization problem are:

$$c_b^E = a_b^E \left(\frac{p_b^E}{P^E}\right)^{-\omega} C^E \tag{1}$$

$$c_a^E = (1-a_b^E)\left(\frac{p_a^E}{P^E}\right)^{-\omega} C^E \tag{2}$$

The market clearing conditions in each country are given by:

$$Y_b^E = c_b^E + c_b^A, \tag{3}$$
$$Y_a^A = c_a^A + c_a^E. \tag{4}$$

And the resource constraints in each country:

$$P^E C^E = P_b^E Y_b^E, \tag{5}$$
$$P^A C^A = P_a^A Y_a^A. \tag{6}$$

[1] Country 'A' faces an equivalent optimization problem.

Under FA, the balanced trade condition for country $E$ is given by:

$$\tau c_a^E = c_b^A, \tag{7}$$

where $\tau = \frac{p_a^E}{p_b^A}$ is defined as the terms of trade. Relatedly, the real exchange rate is defined as $Q = \frac{P^A}{P^E}$.
After log-linearizing eqs. (9) – (15) and solving for $\hat{\tau}$, we obtain the following relationship between the terms of trade and endowment shocks:

$$\hat{\tau} = \frac{\hat{Y}_b^E - \hat{Y}_a^A}{1 - (1-\omega)(a_a^A + a_b^E)}. \tag{8}$$

Notice that the proportionality constant depends on the trade elasticity $\omega$ and the heterogeneous trade share (home bias) parameters of each country $\{a_a^A, a_b^E\}$. Notably, assuming home bias in consumption in both countries, a positive endowment shock in country $E$ is associated with a terms of trade appreciation (i.e., the transmission mechanism of endowment shocks to prices is negative) when the trade elasticity is $\omega < 1 - \frac{1}{(a_a^A + a_b^E)} < 1$.
It is straightforward to obtain an expression for the relationship between the real exchange rate and the terms of trade, $\hat{q} = \hat{p}^A - \hat{p}^E = (a_a^A + a_b^E - 1)\hat{\tau}$.
Similarly, relative consumption is given by $\hat{r}c = \hat{c}^E - \hat{c}^A = \left[\omega(a_a^A + a_b^E) - 1\right]\hat{\tau}$.
Thus, changes in relative consumption are associated with movements in the real exchange rate as follows:

$$\hat{q} = \frac{a_a^A + a_b^E - 1}{\omega(a_a^A + a_b^E) - 1}(\hat{c}_b^E - \hat{c}_a^A) \tag{9}$$

The expression above shows that (with home bias in consumption) the relationship between the real exchange rate and relative consumption is negative when trade is inelastic $\omega < \frac{1}{a_a^A + a_b^E} < 1$.

**A Measure of Risk Sharing.** Heathcote and Perri 2013 point out that the evidence on efficient allocations from exchange rate movements is indirect. Hence, to quantify the implied deviation from full risk sharing I draw on Viani 2010, who proposes a metric for the deviation between complete and incomplete markets allocations. Consider each country's optimality conditions with respect to consumption choice:

$$C^{E^{-\sigma}} = \lambda^E P^E$$

$$C^{A^{-\sigma}} = \lambda^A P^A,$$

where $\lambda_t^i$, $i = \{E, A\}$ is the shadow price of income in country $i$. We take the ratio of the expressions above and log-linearize to obtain:

$$\hat{q} - \sigma(\hat{c}^E - \hat{c}^A) = \hat{\lambda}^E - \hat{\lambda}^A \equiv gap.$$

Viani 2010 defines $gap \equiv \hat{\lambda}^E - \hat{\lambda}^A$ as a measure of cross-border consumption insurance. Under complete markets, households are insured against consumption risk for each and every state of the world, therefore the *gap* is always zero. However, under incomplete markets the *gap* can be different from zero in some states of the world.
From our previous analysis under FA, we can obtain the following expression for the gap in relation to endowment shocks:

$$gap = \hat{q} - \sigma(\hat{c}^E - \hat{c}^A) = \frac{\sigma - 1 + (a_a^A + a_b^E)(1 - \sigma\omega)}{1 - (1 - \omega)(a_a^A + a_b^E)}(\hat{Y}_b^E - \hat{Y}_a^A) \qquad (10)$$

The expression above implies two thresholds for the elasticity of substitution $\omega$. The first threshold, when the gap is largest, is obtained when the denominator is zero and yields $\bar{\omega}_1 = 1 - \frac{1}{a_a^A + a_b^E} < 1$. Notice that this is the upper threshold of $\omega$ for which a positive endowment shock results in a terms of trade appreciation.
The second threshold is obtained when the numerator is zero and it indicates the *efficiency* threshold. That is, when the (incomplete markets) gap corresponds to the prediction of complete markets, namely when $\bar{\omega}_2 = \frac{\sigma + a_a^A + a_b^E - 1}{\sigma(a_a^A + a_b^E)}$. Notice that the efficiency threshold implies full risk sharing ($gap = 0$), even under financial autarky.

**Asymmetric Trade**. Using expressions $(14)-(15)$ we can examine the impact of asymmetric trade on risk sharing for a given (inelastic) trade elasticity of substitution $\omega$. We proceed with a sample parameterization of the financial autarky model $\sigma = 2$, $\omega = 0.5$, $a_b^E = 0.6$. We then examine the effect of changes in the home bias parameter in country $A$, $a_a^A$, on both the risk sharing constant implied by equation (14) and the *gap* in equation (15). The results are shown in Figure C1 (panels A and B).
By isolating the effect of asymmetric trade, we see that as the economy in country $A$ becomes more closed relative to country E, the risk sharing constant becomes more negative and the gap increases.

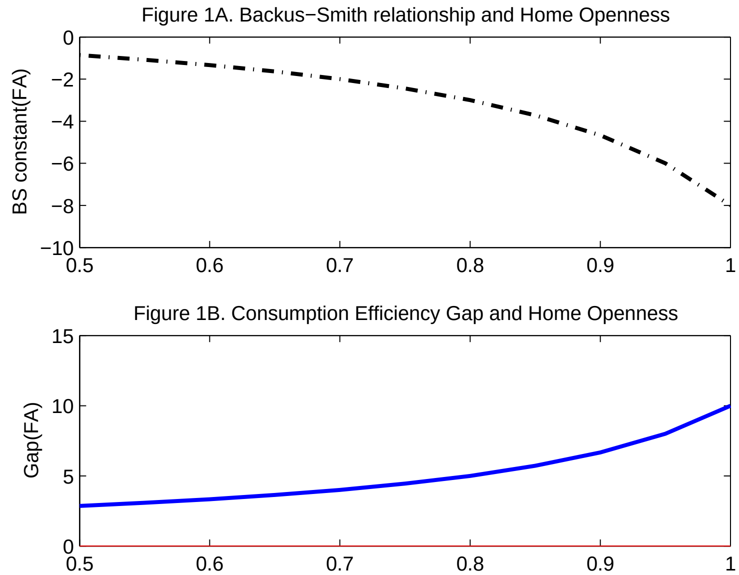


Figure C1. Trade asymmetry and risk sharing

In sum, when country E trades with another country that is relatively less open to trade, this leads to a lowering in the BS statistic and a larger *gap*, hence lower risk sharing.

# References

Heathcote, J. and Perri, F. (2013). Assessing international efficiency. *Handbook of International Economics*, 4(Ch. 9):523–584.

Viani, F. (2010). International financial flows, real exchange rates and cross-border insurance. *Documentos de trabajo del Banco de España*, 38:9–48.