Online Spectrum Auction in Cognitive Radio Networks with Uncertain Activities of Primary Users

Changyan Yi*, Jun Cai†, Gong Zhang‡
*Department of Electrical and Computer Engineering, University of Manitoba, Canada
†Department of Applied Computer Science, University of Winnipeg, Canada
‡Email: {changyan.yi, jun.cai}@umanitoba.ca, gzhang@wrha.mb.ca

Abstract—In this paper, we investigate an online spectrum auction problem in cognitive radio networks with uncertain activities of primary users (PUs). In our framework, a primary base station (PBS), acted as the spectrum auctioneer, leases its under-utilized channels to secondary users (SUs) who request and access spectrum on the fly. Different from most of existing works in online spectrum allocation, we focus on a more practical situation that the auctioneer (or the PBS) has no prior knowledge of PUs’ activities so that its channel states are not static. In order to balance the auction profits from granted SUs’ spectrum requests and the potential penalties caused by incomplete services to PUs, we introduce the idea of virtual spectrum sellers and formulate the problem as an online double spectrum auction. We then propose a novel online admission and pricing mechanism which also considers the reusability of wireless spectrum. Theoretical analyses are provided to prove that our auction algorithm satisfies all desired economic properties in terms of budget-balance, individual rationality and truthfulness. Simulation results show that our proposed auction algorithm can increase the utility of the PBS, enhance spectrum utilization and achieve better satisfaction for SUs compared to counterparts.

I. INTRODUCTION

Current spectrum regulatory policy encounters an enormous challenge in spectrum efficiency with dramatically growing demand of spectrum from newly developed wireless equipments and applications. As a promising approach to address this issue, cognitive radio (CR) [1], [2] has been proposed, which enables secondary users (SUs) to dynamically exploit the under-utilized licensed spectrum. However, since the implementation of CR requires the cooperation of spectrum owners or primary users (PUs), it is crucial to provide incentives for PUs to release their unused radio resources. From the perspective of engineering economics, auction-based spectrum sharing has been regarded as a fair and efficient method, by which spectrum owners could obtain potential economic profits by leasing their redundant spectrum to SUs who are willing to pay.

Most of existing researches focused on traditional single-round offline spectrum auction [3]–[6]. For instance, Kash et al. in [3] presented a strategy-proof and scalable spectrum auction algorithm for the allocation of spectrum rights among both sharers and exclusive-use bidders. Wu et al. in [5] studied a sealed-bid reserve auction mechanism and brought forward an illustration for multi-band spectrum buyers. Though single-round auctions could be processed periodically, they are not suitable for continuous spectrum auction where SUs request spectrum on the fly with potential tolerable delays. Online auction [7] is a technique to cope with this issue where each spectrum buyer could submit its request at any arrival time for a service that could be accomplished within its tolerable delay. In [8], the authors proposed a truthful online spectrum auction framework for CR networks that allocated spectrum efficiently by exploring both spatial and time reusability while resisting bidders from misreporting their valuations and time requirements. Sodagari et al. in [9] investigated a truthful mechanism for expiring spectrum sharing where the property of collusion-resistance was proved in details. However, almost all the works on online spectrum auctions presumed that the number of licensed channels for leasing is static, and did not take into account the potential uncertainty on channel availability caused by uncertain and frequent PUs’ spectrum usages.

In this paper, we readdress the online spectrum auction problem in CR networks by considering uncertain activities of PUs. In our system model, there is a primary base station (PBS) who owns multiple licensed radio channels and is responsible to protect PUs’ spectrum usages. At the same time, the PBS also runs an online auction to lease its idle channels to SUs who request and access spectrum on the fly. By considering a practical situation where the PBS has no a priori information of PUs’ uncertain activities, the PBS may suffer a great penalty if the PBS is only eager to improve its potential auction revenue while ignoring its own PUs’ spectrum usages. On the other hand, if the PBS reserves channels excessively to completely protect its own PUs, it may lose economic profits from the spectrum auction. To balance the penalties introduced by incomplete service for PUs and the auction profits from granted SUs’ spectrum requests, we propose a virtual online double spectrum auction (VIOLET) algorithm. In VIOLET, virtual spectrum sellers are defined to describe the channel uncertainties. Moreover, the well designed online admission and pricing mechanism of VIOLET can ensure non-deficit utility of the PBS while resisting mendacious behaviours from selfish SUs. Theoretical analyses prove that VIOLET is economic robust in terms of budget-balance, individual rationality and truthfulness. In addition, simulation results show that VIOLET can improve the utility of the PBS, enhance spectrum utilization and achieve better satisfaction of SUs.
To our best knowledge, we are the first to address online double auction in dynamic spectrum allocation without a priori information of both SUs and PUs.

The rest of this paper is organized as follows: Section II introduces the system model. Section III describes the proposed VIOLET algorithm in details with its design goals, admission and pricing mechanism. Section IV provides solid theoretical analyses for economic properties. Simulation results are presented in Section V. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

Consider a CR network with one PBS and \( m \) subscribed PUs registered with it. Each PU demands exclusive-usage of one licensed channel so that the PBS owns the license of \( m \) homogeneous orthogonal spectrum channels and is responsible for serving all its PUs’ communication requests. Obviously, the PBS could serve all its registered PUs simultaneously and there is no competition among PUs. However, the activities of PUs are uncertain, which means that any PU may declare its spectrum request at any time for any long period. Since PUs have already signed a service contract with the PBS, they cannot suffer any delay. Moreover, if a request from PU \( j \) is not served, the PBS would be punished by a predetermined penalty \( \phi_j \). In CR networks, PUs have higher channel access priority than SUs and their services are protected due to the contracts pre-signed with the PBS. Thus, it is reasonable to assume that information provided by PUs are always truthful.

The network also includes some SUs who are willing to buy the usage of idle spectrum at any time. Assume that SUs \( S = \{s_1, s_2, \ldots \} \) will request spectrum usage on the fly. Since in wireless networks, SUs may be located in different geometric areas, this location-dependent characteristic allows spectrum spatial reuse among time overlapped SUs’ requests. To capture the spectrum spatial reuse, we model the SU network as a conflict graph \( \mathcal{H} = (\mathcal{S}, \mathcal{E}) \) by applying existed methods [10], [11], where \( \mathcal{S} \) is the vertex set corresponding to the requests of SUs and two SUs \( s_i \) and \( s_k \) form an edge \( (s_i, s_k) \in \mathcal{E} \) if and only if they cannot access the same channel simultaneously. We further limit our discussion on the scenario that each SU only demands a single time-frequency chunk in each request, i.e., a single time slot from one channel. Such scenario has been widely applied for file transmissions (such as HTTP/FTP) [9], where discrete radio resources are valuable. Let \( e_1, e_2, \ldots, e_i, \ldots \) be the sequence of requests over \( T \). Each request is expressed as \( e_i = (s_i, a_i, d_i, v_i) \), called the profile of request from SU \( i \), which indicates that SU \( s_i \) declares a spectrum request at arrival time \( a_i \in T \) for a service within \( [a_i, d_i] \), where \( d_i \in T \) is the deadline for fulfilling this request and \( v_i \in \mathbb{R}^+ \) represents its valuation for receiving the service. If \( d_i - a_i > 1 \), some delay tolerance exists for the service of \( e_i \). Note that each SU may request multiple non-overlapped spectrum usages. However, since there is no need to differentiate SUs if their conflict graph is already known, for notation simplicity, we omit index \( s_i \) in the following context and use \( e_i = (a_i, d_i, v_i) \) to denote the \( i \)-th request from SUs.

Let the PBS be the spectrum auctioneer and its objective is to improve spectrum utilization and its own overall utility. We focus on the spectrum allocation during a long time period \( T = [0, T] \) which consists of \( T \) normalized time slots.

III. VIRTUAL ONLINE DOUBLE SPECTRUM AUCTION (VIOLET) ALGORITHM

In this section, we first introduce the idea of virtual seller and explain how the channel uncertainty can be represented as the online activities of virtual sellers. Introducing virtual sellers provides us a possibility to formulate the problem to an online double spectrum auction. Then, we describe the admission and pricing mechanism of VIOLET.

A. Virtual Spectrum Sellers

In order for the PBS to make allocation decisions for SUs while ensure itself a non-deficit utility, we introduce a concept of virtual spectrum sellers in the auction based on the uncertain activities of PUs. Specifically, the PBS creates \( I(t) \) virtual sellers at any time instant \( t \in T \), where \( I(t) \) equals the number of idle channels at the end of \( t - 1 \). The generation of virtual sellers is as follows.

i) For a request of PU arrived at \( t \), the PBS generates a corresponding virtual seller \( j \) with \( c_j = \{ (a_j = t, d_j = t + 1, v_j = \phi_j) \} \), which indicates that the seller who arrives at \( a_j \) has no patience (departs in one time slot) and wants to sell its resource at an asking price of \( \phi_j \).

Note that setting \( d_j = a_j + 1 \) only means that the PU’s request has no delay tolerance. In fact, the PU can request multiple time slots. Once it wins the game, it will stay in the system till its requested service is finished.

ii) If the number of newly arrived PUs, \( A(t) \), is less than \( I(t) \), the PBS automatically adds \( I(t) - A(t) \) virtual sellers, all with asking price equal to 0, i.e., \( c_j = \{ (a_j = t, d_j = t + 1, v_j = 0) \} \).

Note that, unlike traditional auctions, sellers in our case are not real, which means that they do not have real incomes. In other words, if a virtual seller wins the auction, its income (which is actually the penalty on the PBS) will be exactly its asking price.

B. Design Goals

Let \( e^b \) and \( e^t \) denote the set of requests from SUs (buyers) and virtual sellers in \( t \), respectively. We further define \( e = (e^1, \ldots, e^t) \) and \( e^c = (e^1, \ldots, e^t, e^t) \) denote the complete request profile over \( T \). Since SUs are self-interested and their requests are private information, SU \( i \) may misreport its request, i.e., \( \hat{e}_i = (\hat{a}_i, \hat{d}_i, \hat{v}_i) \neq e_i \), if it could benefit from such behavior. Similar as most of the online mechanisms [8], [9], we assume that there is no misreport of either early-arrival or later-departure in the system. In practice, reporting \( \hat{a}_i < a_i \) or \( \hat{d}_i > d_i \) may probably lead to a service beyond the range of \( [a_i, d_i] \), which is not expected by the SU. In addition, all SUs’ requests are assumed to have a bounded patience, i.e., \( d_i \leq a_i + \Delta \) and \( \Delta \neq \infty \).

For bidder (both buyers and sellers) profile \( \theta = e \cup e^c \), let \( \theta^t^- \) represent the profile with arrival time no later than \( t \).
An online double spectrum auction mechanism, \( M \triangleq (x, p) \), defines an allocation scheme \( x = \{x^t\}_{t \in T} \) and a payment policy \( p = \{p^t\}_{t \in T} \), where \( x^t \) and \( p^t \) denote the allocation and payment vector at each time slot \( t \), respectively. For each bidder \( k \) with its profile \( \theta_k \in \Theta \), we further define \( x^*_k(\theta^t-\cdot) \in \{0, 1\} \) to indicate whether bidder \( k \) wins no later than \( t \) and \( p^*_k(\theta^t-\cdot) \in \mathbb{R} \) to indicate its payment. Note that \( p^*_k(\theta^t-\cdot) \geq 0 \) if bidder \( k \) is a buyer, i.e., \( \theta_k \in \mathbf{e}^{-t} \), and \( p^*_k(\theta^t-\cdot) \leq 0 \) if bidder \( k \) is a seller, i.e., \( \theta_k \in \mathbf{e}^t \). For a feasible mechanism, it must satisfy the condition that \( x^*_k(\theta^t-\cdot) = 1 \) in at most one slot \( t \in [a_k, d_k] \) and zero for other values of \( t \).

Now, we can define the desired economic properties in terms of budget-balanced, individual rationality and truthfulness.

**Definition 3.1 (Budget-Balance):** \( M \triangleq (x, p) \) is budget-balanced if the utility of the PBS is always non-deficit, i.e.,

\[
U_{pbs}^t = \sum_{\theta_k \in \Theta} \sum_{t' \in \{a_k, \min(t, d_k)\}} p^*_k(\theta^{t'-\cdot}) \geq 0, \quad \forall t' \in T. \tag{1}
\]

This property ensures that the PBS could always benefit from running the auction, even though it suffers potential penalties from its own SUs.

Since individual rationality and truthfulness are only required for SUs, we let \( x_i(\mathbf{e}) = \sum_{t' \in [a_i, d_i]} x_i^t(\mathbf{e}^{t'-\cdot}) \) and \( p_i(\mathbf{e}) \) indicate whether \( e_i \) wins or not and its payment, respectively.

**Definition 3.2 (Individual Rationality):** \( M \triangleq (x, p) \) is individually rational for all SUs’ requests, if no \( e_i \) pays more than its valuation, i.e.,

\[
v_i(x(\mathbf{e})) - p_i(\mathbf{e}) \geq 0, \quad \forall i \in \{i | \theta_i \in \mathbf{e}\} \tag{2}
\]

where \( v_i(x(\mathbf{e})) \) represents the valuation of \( e_i \), given the allocation \( x(\mathbf{e}) \), i.e., \( v_i(x(\mathbf{e})) = v_i \) if \( e_i \) wins in \( x(\mathbf{e}) \) and \( v_i(x(\mathbf{e})) = 0 \), otherwise. With this property, the utility of SUs can be always non-negative which provides them incentives to participate.

Before introducing the definition of truthfulness, we define \( \mathbf{e}^{-i} \in \mathbf{e} \) as the set of other requests except \( e_i \), and \( \mathbf{e}(i) \) as the set of potential misreports of \( e_i \).

**Definition 3.3 (Truthfulness):** \( M \triangleq (x, p) \) is incentively compatible or truthful if no SU can improve its utility by misreporting its type, i.e.,

\[
v_i(x(\mathbf{e}_i, \mathbf{e}^{-i})) - p_i(\mathbf{e}_i, \mathbf{e}^{-i}) \geq v_i(x(\mathbf{\hat{e}}_i, \mathbf{e}^{-i})) - p_i(\mathbf{\hat{e}}_i, \mathbf{e}^{-i}),
\]

where \( \mathbf{\hat{e}}_i \in \mathbf{e}(i) \), \( \forall i \in \{i | \theta_i \in \mathbf{e}\} \). \tag{3}

This property is essential for a robust auction equilibrium. It resists market manipulation and ensures auction efficiency and fairness.

**C. Admission and Pricing Mechanism \( \mathcal{M}_e \)**

In the following, we introduce the detailed steps of a novel online mechanism \( \mathcal{M}_e \), which considers the reusability of wireless spectrum and satisfies all required economic properties.

1) **Grouping SUs’ Requests:** At any time instant \( t \), the PBS could group the outstanding SUs’ spectrum requests based on the pre-determined conflict graph \( \mathcal{H} \). For SUs that do not interfere with each other, their requests are grouped into the same group and each of them can be assigned the same spectrum chunks. Such process is equivalent to finding independent sets of the conflict graph and is processed privately by the PBS. Specifically, the PBS could recursively select a node in current conflict graph and include it to the set, eliminate the chosen node and its neighbors, and update the topology of the remaining nodes.

Let \( \xi_1^t, \xi_2^t, \ldots, \xi_L(t) \) denote the \( L(t) \) buyers’ groups formed at \( t \). We can simply regard each \( \xi_i^t \) as a super buyer with \( |\xi_i^t| \) non-conflict members. Then, the group bid \( g_i^t \) can be calculated as

\[
g_i^t = \min\{\hat{v}_i|e_i^t \in \xi_i^t\} \times |\xi_i^t| \tag{4}\]

where \( \min\{\hat{v}_i|e_i^t \in \xi_i^t\} \) represents the minimum reporting valuation of a request in group \( \xi_i^t \).

2) **Myopic Matching and Pricing:** For a specific time slot \( t \), we have \( I(t) \) virtual sellers. For clarity, we use \( \beta_i^t \in \{v_j|\theta_j \in \mathbf{e}^t\} \) to denote the asking price of virtual seller \( j \) at \( t \). Inspired by static McAfee [12] matching rule, the PBS sorts all buyers’ group bids and sellers’ asking prices collected at \( t \) in a non-increasing and a non-decreasing order, respectively, i.e.,

\[
g_1^t \geq g_2^t \geq \cdots \geq g_{I(t)}^t \leq \beta_1^t \leq \beta_2^t \leq \cdots \leq \beta_{I(t)}^t \tag{5}
\]

Now, match the above two orders one by one, and let \( r \) index the last profitable pair, i.e.,

\[
r = \arg \max_{r' \leq \min\{I(t), I(t)\}} g_1^t - \beta_r^t \geq 0 \tag{5}
\]

Then, the first \( r-1 \) buyer groups will win the auction and the first \( r-1 \) virtual sellers will be traded at \( t \). On the other hand, the rest of virtual sellers will lose and their corresponding number of channels will be reserved for PUs. To guarantee myopic truthfulness, each winning buyer group \( \xi_i^t \) will be charged by the \( r \)-th buyer group’s bid \( g_r^t \) (the highest losing bid), and such group payment is shared equally among all SUs’ requests in group \( \xi_i^t \), i.e.,

\[
p^t_i = g_r^t/|\xi_i^t|, \quad \forall e_i \in \xi_i^t. \tag{6}
\]

Any losing SUs’ request does not need to pay and no virtual seller is paid with real profit. However, the PBS would be penalized for trading virtual sellers with their asking prices. Therefore, the myopic utility of the PBS can be expressed as

\[
U_{myopic}^t = \sum_{j=1}^{r-1} (g_j^t - \beta_j^t) \tag{7}
\]

3) **Online Payment Calculation:** In order to resist both bid- and time-based cheating from SUs while maintain budget balance for the PBS, we propose a feasible online pricing scheme for any SU’s request \( e_i \) as follows.

- **Upon arrival:** Consider the myopic double auction in all its possible early arrival time \( t' \in [d_i - \Delta, a_i - 1] \) with its reported bid, where \( \Delta \) is our defined bounded patience. If it would lose in all its early arrival time, we set its admission price \( \eta(a_i, d_i, e_{-i}) = 0 \). Otherwise,

\[
\eta(a_i, d_i, e_{-i}) = \max\{p_i^t|t' \in [d_i - \Delta, a_i - 1]\} \tag{8}
\]

where \( p_i^t \) indicates the myopic pricing that such request has to pay at \( t' \).
During active period: For any $t$ from $\tilde{a}_i$ to $\tilde{d}_i$, if the request wins the myopic auction at $t$, it will be selected as a winner and its final payment is calculated as

$$p_i(\tilde{a}_i, \tilde{d}_i, e_{-i})) = \max(p_i, \eta(\tilde{a}_i, \tilde{d}_i, e_{-i})) \quad (9)$$

If the request cannot win in any time $t \in [\tilde{a}_i, \tilde{d}_i]$, its payment is set as $0$.

Corollary 3.1: The proposed online double spectrum auction mechanism, $\mathcal{M}_c$, can be reduced to a sequence of myopic TRUST algorithm [13] if all SUs’ spectrum requests are impatient to any delay, i.e., $\Delta = 1$.

IV. PROOF OF ECONOMIC PROPERTIES

In this section, we prove that our auction mechanism $\mathcal{M}_c$ satisfies all desired economic properties through theoretical analyses.

Theorem 4.1: $\mathcal{M}_c$ is budget-balanced for the PBS and individually rational for all the SUs’ requests.

Proof: According to the myopic matching rule, we can simply observe that only the bid-ask pair with bid greater than ask would be selected to trade at a time $t$. Moreover, since the PBS does not need to pay for winning virtual sellers and its penalty equals the sum of exactly their asking prices, the myopic utility of the PBS, $U_{\text{myopic}}^t \geq 0$. In addition, the final payment of each winning SU’s request is calculated as the maximum payment it has made if it could win in any time instant from its possible earliest arrival to its winning instant. Therefore, the utility of the PBS at any time $t$ is even larger than its myopic utility, i.e.,

$$U_{\text{PBS}}^t \geq U_{\text{myopic}}^t \geq 0, \quad \forall t \in T. \quad (10)$$

Hence, $\mathcal{M}_c$ ensures budget-balance for the PBS.

Individual rationality can be immediately proved from the pricing scheme. First, a SU has to pay only if its request $e_i$ could win at $t \in [a_i, d_i]$. Even though the payment $p_i(e)$ is not simply equal to the market clearing price at $t$, such payment must be calculated from one of the myopic double auction during $[d_i - \Delta, t]$, in which $e_i$ could win. Based on our myopic pricing rule, each buyer group declares its group bid based on the minimum individual valuation of its members and if wins the auction, it pays the highest bid from losing groups. Thus, we always have

$$v_i \geq p_i(e) \geq p_i, \quad \exists r \in [d_i - \Delta, t]. \quad (11)$$

Hence, $\mathcal{M}_c$ guarantees individual rationality.

In order to prove the truthfulness of $\mathcal{M}_c$ for all SUs’ requests, we first examine the following prerequisite technical lemmas.

Lemma 4.1: At a single time slot $t \in [a_i, d_i]$, if a SU’s request $e_i$ wins in $\mathcal{M}_c$, then for fixed $a_i$, $d_i$ and $e_{-i}$, the myopic payment, $p_i$, is independent of its bid value $v_i$.

Proof: Proving the myopic value-independency is equivalent to proving that if $e_i$ wins at $t$ by bidding $\hat{v}_i$ or $\tilde{v}_i$, the myopic payment charged from $e_i$ is the same for both bid values. Let $\xi_i$ denote the group that $e_i$ belongs to and $g_i^f$, $d_i^f$ be the bids of $\xi_i$ when $e_i$ bids $\hat{v}_i$ and $\tilde{v}_i$, respectively. If $e_i$ wins with both groups, the market clearing price of $\xi_i$ is determined by the same highest losing group bid, $g_i^f$, ranked after $g_i^f$ and $d_i^f$. Furthermore, the size $|\xi_i|$ also keeps the same for bidding $\hat{v}_i$ or $\tilde{v}_i$ from $e_i$. Therefore, our claim holds since $p_i = g_i^f/|\xi_i|$.

Lemma 4.2: No matter a SU’s request $e_i$ wins or loses in $\mathcal{M}_c$, for fixed $a_i$, $d_i$ and $e_{-i}$, reporting $\hat{v}_i > v_i$ for which it can win the auction will lead to a payment, $\hat{p}_i(e)$, larger or equal to $p_i(e)$. More seriously, such misreporting may also break the property of individual rationality.

Proof: We separate our discussions for two cases, i.e., whether or not $e_i$ could win with truthful value $v_i$. C1: If $e_i$ wins by bidding $v_i$, misreporting $\hat{v}_i \geq v_i$ would not change the myopic payment, $p_i$, at its trading time $t$ (Lemma 4.1). However, since $\hat{v}_i$ may result in more winnings during $[d_i - \Delta, a_i - 1]$, the admission price $\eta(a_i, d_i, e_{-i})$ of $e_i$ would monotonically increase according to (8). With the calculation of final payment in (9), we can directly find that $\hat{p}_i(e) \geq p_i(e)$. C2: If $e_i$ loses by bidding $v_i$, it also means that $e_i$ cannot win any myopic double auction during $[a_i, d_i]$. Suppose that $\exists \hat{\tau} \in [a_i, d_i]$ when $e_i$ could win with $\hat{v}_i$. Let $g_i^f$ and $d_i^f$ be the bid of $e_i$’s group $\xi_i$ when $e_i$ reports $v_i$ and $\hat{v}_i$ at $\hat{\tau}$, respectively. Since the auction results of $e_i$ is changed by bidding $\hat{v}_i > v_i$, the bid of $e_i$ must be the lowest one in $\xi_i$, i.e., $v_i = g_i^f/|\xi_i|$. Moreover, since $e_i$ loses by bidding $v_i$ while winning by bidding $\hat{v}_i$, we have $g_i^f \leq \kappa^f \leq d_i^f$, where $\kappa^f$ is the market clearing price at $\hat{\tau}$. Therefore, the utility of $e_i$ with bids $\hat{v}_i$ is $v_i - \kappa^f/|\xi_i| \leq 0$. Consequently, with C1 and C2, Lemma 4.2 is proved.

With Lemma 4.1 and Lemma 4.2, we could further examine that $\mathcal{M}_c$ satisfies the price-based characterization [14] which is essential for establishing truthfulness in online auctions.

Definition 4.1 (price-based characterization [14]): An online auction is price-based if there is a value-independent pricing scheme such that $e_i$ wins if and only if $p_i(a_i, d_i, e_{-i}) \leq v_i$ and the payment of a winning $e_i$, $p_i(e) = p_i(a_i, d_i, e_{-i})$.

Lemma 4.3: $\mathcal{M}_c$ is a price-based mechanism.

Proof: From Lemma 4.1, we know that if a SU’s request $e_i$ wins at a time $t \in [a_i, d_i]$, then for fixed $a_i$, $d_i$ and $e_{-i}$, its myopic payment $p_i$ is independent of its required value $v_i$. Moreover, we know that such payment must be less than or equal to the minimum bid value across all SUs’ requests that will be granted at $t$. From Lemma 4.2, we also see that such payment must be greater than all bid values from losing requests. In conclusion, there is a value-independent price for trading at $t$, which is the minimal value that a winning SU’s request could have bid at that time instant. Finally, note that for fixed $[a_i, d_i]$ and fixed $e_i$, the request $e_i$ would always win at the same time with its bid value greater than the myopic payment. Hence, $p_i(e) = p_i(a_i, d_i, e_{-i})$ as required and Lemma 4.3 holds.

Furthermore, we prove that no SU’s request could benefit from potentially misreporting $\tilde{d}_i > a_i$ or $d_i < d_i$ through next lemma.

Definition 4.4: The payment of a winning $e_i$ in $\mathcal{M}_c$ is monotonic increasing that $p_i(a_i, d_i, e_{-i}) \leq p_i(\tilde{a}_i, d_i, e_{-i})$ for

\[
p_i(\hat{a}_i, \hat{d}_i, e_i) = \max\{p_i^* | r_1 \in [\hat{d}_i - \Delta, t]\}
\]
\[
p_i(a_i, d_i, e_i) = \max\{p_i^* | r_2 \in [d_i - \Delta, t]\}
\]

- \(\hat{a}_i, \hat{d}_i \subseteq [a_i, d_i]\), and no \(e_i\) can win in an earlier time instant by reporting \([\hat{a}_i, \hat{d}_i] \subseteq [a_i, d_i]\).

\textbf{Proof:} According to the online pricing scheme in (8) and (9), it could be easily derived that, for any winning \(e_i\), we have

\[
p_i(\hat{a}_i, \hat{d}_i, e_i) = \max\{p_i^* | t' \in [\hat{d}_i - \Delta, t]\}
\]

where \(t\) denotes its trading time.

As shown in Fig. 1, the payment of the winning \(e_i\) is independent of its arrival time and increased with an earlier departure since

\[
\max\{p_i^* | r_1 \in [\hat{d}_i - \Delta, t]\} \geq \max\{p_i^* | r_2 \in [d_i - \Delta, t]\}
\]

for any \(\hat{d}_i < d_i\). Thus, it only remains to prove that reporting \([\hat{a}_i, \hat{d}_i] \subseteq [a_i, d_i]\) cannot bring an earlier winning. Notice that no SU’s request would like to delay its winning time because the payment is obviously non-decreasing with \(t\) according to (12). Furthermore, reporting an earlier departure time cannot change any auction results if \(t \leq d_i\). Hence, we only need to consider the case that \(e_i\) may misreport \(\hat{a}_i > a_i\). Since a later arrival has no effect on the payment due to the independency, intentionally delaying arrival will not lead to an early winning for \(e_i\), but in turn, may possibly make it miss opportunities of early winnings. In summary, Lemma 4.4 is guaranteed.

Taken all the above lemmas (Lemma 4.1-Lemma 4.4) together, we could conclude that \(M_c\) is implemented by a price-based auction with value-independent payment scheme, and the payment of each winning \(e_i\) is monotonically increasing with i) reporting later arrival, i.e., \(\hat{a}_i > a_i\), given \(\hat{d}_i = d_i\); ii) reporting earlier departure, i.e., \(\hat{d}_i < d_i\), given \(\hat{a}_i = a_i\); or iii) the combination of them. According to [15], we obtain the following theorem.

\textbf{Theorem 4.2:} \(M_c\) is truthful for all the SUs’ requests.

\section*{V. Simulation Results}

In this section, we conduct simulations to evaluate our proposed \textbf{VIOLET} algorithm. For comparison purpose, \textbf{TOPAZ} [8] is simulated as the benchmark. Since no prior works consider channel uncertainty of the PBS in online spectrum auctions, for a fair comparison, we also simulate a modified \textbf{TOPAZ}, called \textbf{M-TOPAZ}, by simply adding a procedure to examine the number of available auctioned channels at the beginning of each time slot so that the PUs’ services can be completely protected. All simulation parameters are shown in Table I and all results are based on the average over 20 runs.

Fig. 2 shows cumulative utilities of the PBS at each time slot. Since \textbf{TOPAZ} only considers one-sided spectrum auction to SU users and ignore the potential PUs’ spectrum usages, the utility of the PBS has a large variance (up to 327.4\%) due to the penalty from the uncertain activities of PUs. As a result, the property of budget-balance cannot always be guaranteed. On the other hand, the PBS suffers zero penalty with \textbf{M-TOPAZ} and obtains extra revenue from leasing its unused channels. \textbf{VIOLET} better balances the tradeoff between auction revenue and penalty, so that it achieves the best performance.

Figs. 3 and 4 demonstrate the impact of increasing PUs’ arrival rate and the number of primary channels, respectively. It is shown that the overall utility of the PBS in \textbf{VIOLET} decreases with the increase of PUs’ arrival rate. However, its curve is descended slower than those of \textbf{TOPAZ} and \textbf{M-TOPAZ}. It is because the increase of PUs’ spectrum usage results in a larger penalty on \textbf{TOPAZ} and less auction revenue on \textbf{M-TOPAZ}, respectively. In addition, Fig. 4 indicates that \textbf{VIOLET} can perform better than \textbf{M-TOPAZ} on the PBS’s utility with the increase of the total number of channels.

Spectrum utilizations are compared in Fig. 5, which is calculated as the ratio between the total spectrum usage of winning users (both SU users and PU users) and the total spectrum resources \(m \times T\). Without auction, such ratio is only determined by the activities of PUs. Enabling spectrum auction significantly improves the utilization due to the spectrum reuse among non-


\begin{table}[h]
\centering
\caption{Simulation Settings}
\begin{tabular}{|l|l|}
\hline
\textbf{Parameters} & \textbf{Value} \\
\hline
\hline
Area size & \(200 \times 200\) \\
Conflict distance & 30 \\
Number of time slots & 50 \\
Bounded tolerance & 3 \\
\hline
\hline
\textbf{Parameters} & \textbf{Model} \\
\hline
Bid valuations & \(\text{Uniform}[0, 1]\) \\
PUs’ penalties & \(\text{Uniform}[0.5, 1]\) \\
Interarrival time of PUs & \(\text{Exponential}\) \\
Service time of PUs & \(\text{Exponential}\) \\
\hline
\hline
\textbf{Parameters} & \textbf{Model} \\
\hline
Number of channels & \([1, 10], 5\) \\
Number of SU users’ requests & \([100, 500], 300\) \\
PUs’ arrival rate & \([1, 3], 2\) \\
PUs’ service rate & \([0, 1], 0.5\) \\
\hline
\end{tabular}
\end{table}
![Graph 1](image1.png)

![Graph 2](image2.png)

![Graph 3](image3.png)

![Graph 4](image4.png)

![Graph 5](image5.png)

In this paper, an online spectrum auction with uncertain activities of PUs has been studied for CR networks. In order to guarantee a non-deficit utility of the PBS so as to provide it economic incentives to participate in the auction, we propose a virtual online double spectrum auction algorithm, called VIOLET, in which the channel uncertainties are represented by the online activities of virtual spectrum sellers. Theoretical analyses prove that VIOLET can ensure non-negative utilities for all users, and resist untruthful behaviors from SUs. Simulation results indicate that VIOLET can enhance the spectrum allocation efficiency in terms of spectrum utilization, utility of the auctioneer and buyers’ satisfaction ratio.

## VI. Conclusions

VI. CONCLUSIONS

In this paper, an online spectrum auction with uncertain activities of PUs has been studied for CR networks. In order to guarantee a non-deficit utility of the PBS so as to provide it economic incentives to participate in the auction, we propose a virtual online double spectrum auction algorithm, called VIOLET, in which the channel uncertainties are represented by the online activities of virtual spectrum sellers. Theoretical analyses prove that VIOLET can ensure non-negative utilities for all users, and resist untruthful behaviors from SUs. Simulation results indicate that VIOLET can enhance the spectrum allocation efficiency in terms of spectrum utilization, utility of the auctioneer and buyers’ satisfaction ratio.

## REFERENCES


