# Canadian Abstract Harmonic Analysis Symposium 2017

In Honour of the Retirement of Professor Fereidoun Ghahramani

Tuesday May 23		Wednesday May 24	
8:40 - 9:00	Registration	9:00 - 9:50	H. G. Dales
9:00 - 9:15	Welcome	10:00 - 10:25	Joel Feinstein
9:15 - 10:05	Anthony To-Ming Lau	10:30 - 11:00	Coffee Break
10:15 - 10:45	Coffee Break	11:00 - 11:25	Keith Taylor
10:45 - 11:10	Mehdi Monfared	11:30 - 11:55	Christopher Ramsey
11:15 - 11:40	Davood Malekzadeh	12:00 - 1:45	Lunch
11:45 - 12:10	Prachi Loliencar	1:45 - 2:35	Richard J. Loy
12:15 - 2:00	Lunch	2:45 - 3:10	Eman Aldabbas
2:00 - 2:50	Yemon Choi	3:15 - 3:45	Coffee Break
3:00 - 3:25	Nico Spronk	3:45 - 4:10	Safoura Zadeh
3:30 - 4:00	Coffee Break	4:15 - 4:40	Varvara Shepelska
4:00 - 4:25	Jason Crann	6:00 - 9:00	Banquet
4:30 - 4:55	Matthew Wiersma		

## Schedule

Thursday May 25		
9:00 - 9:50	Sandy Grabiner	
10:00 - 10:25	Mahya Ghandehari	
10:30 - 11:00	Coffee Break	
11:00-11:25	Ross Stokke	
11:30-12:20	Matthias Neufang	

All talks will be held in the Robert B. Schultz Lecture Theatre in St. John's College at the University of Manitoba, Fort Garry Campus. A link to the campus map can be found on the conference website.

### **Plenary Speakers**

#### Speaker: **Yemon Choi** (Lancaster University)

Title: Some results on direct finiteness of group algebras

Abstract: A unital ring is said to be directly finite if every left-invertible element is automatically invertible; one can extend this definition to the non-unital setting. If G is a discrete group, then its group von Neumann algebra VN(G) is directly finite, and hence we may deduce that its  $\ell^1$ -group algebra is directly finite. In this talk, I will explain how the proof works, and then discuss what can be said for convolution algebras arising from unimodular, locally compact groups. If time permits, I will present some partial results and open questions concerning the algebras of p-pseudofunctions and p-convolvers.

Speaker: H. G. Dales (Lancaster University) Title: Banach function algebras and BSE norms

Abstract: Joint work with Ali Ülger, Istanbul

Let A be a Banach function algebra on its character space  $K = \Phi_A$ ; an evaluation character at x is  $\varepsilon_x$ , and L(A) is the linear span of  $\{\varepsilon_x : x \in K\}$  as a subset of A'. We set

 $||f||_{\mathsf{BSE}} = \sup\{|\langle f, \lambda \rangle| : \lambda \in L(A)_{[1]}\} \quad (f \in C^b(K)),$ 

and  $C_{BSE}(A) = \{ f \in C^b(K) : \|f\|_{BSE} < \infty \}.$ Clearly  $K \subset L(A)_{[1]} \subset A'_{[1]}$ , and so  $\|f\|_K \le \|f\|_{BSE} \le \|f\| (f \in A)$ . In fact,  $\|\cdot\|_{BSE}$  is an algebra norm on A.

**Definition** A Banach function algebra A has a BSE norm if there is a constant C > 0 such that  $||f|| \leq C ||f||_{BSE}$   $(f \in A)$ . The algebra A is a **BSE algebra** whenever  $\mathcal{M}(A) = C_{BSE}(A)$ , where  $\mathcal{M}(A)$  is the multiplier algebra of A.

We shall discuss when a variety of Banach function algebras, mainly related to locally compact groups, are BSE algebras and when they have a BSE norm.

Related ideas involve the Arens regularity of A and when A is an ideal in its bidual.

#### Speaker: Sandy Grabiner (Pomona College)

#### Title: What do we know about weighted convolution algebras on $\mathbb{R}^+$ ?

Abstract: I will give a survey of results about weighted convolution algebras  $L^1(\mathbb{R}^+,\omega)$ on the non-negative real numbers, particularly the case of radical Banach algebras. The modern study of such algebras begins with a fundamental paper circulated by Graham Allan in the 1970s. I will emphasize the study of homomorphisms, particularly a class of well behaved homomorphisms called standard homomorphisms. Much of the work on these homomorphisms is joint work with Fereidoun Ghahramani.

#### Speaker: Anthony To-Ming Lau (University of Alberta)

Title: Fixed point sets for semigroup of mappings in harmonic analysis

Abstract: Let K be a non-empty compact convex subset of a separated locally convex space and  $S = \{Ts : s \in S\}$  be a semigroup of continuous mappings from K into K. In this talk I shall discuss some recent results on the following general questions related to abstract harmonic analysis. :

Problem 1. When is the fixed point set of S:

$$F(S) = \{x \in K; Tsx = x \text{ for all } x \in S\}$$

non-empty?

Problem 2. Suppose we know that F(S) is non-empty, what can we say about elements in F(S)? or spanF(S)?

### Speaker: Richard J Loy (Australian National University)

Title: Approximate amenability – Hows, Whys and Wherefores

Abstract: A broad overview of the topic will be given, from its inception in 2000 to the current state of play.

#### Speaker Matthias Neufang (Carleton University)

Title: Topological centres, module maps, and invariant means

Abstract: We present an overview of some of our work on topological centres, as well as related topics such as the structure of module maps on the dual of a Banach algebra A, and invariant means. Our results include solutions to several problems raised in the literature:

- CsiszárÕs conjecture (1971) on the topological centre of  $LUC(G)^*$  for general topological groups (solved for all separable G, jointly with Ferri);
- the Ghahramani-Lau conjecture (1994/95) on the topological centre of the measure algebra (jointly with Losert-Pachl-Steprāns);
- questions of Lau-Ülger (1996, 2014) on the structure of module maps on  $A^*$  stemming from topological centre elements (jointly with Hu-Ruan), and of natural and invariant projections on  $A^*$ ;
- a question of Dales (2007) on small dtc sets, i.e., determining for the topological centre (jointly with Ferri-Pachl);
- the Farhadi-Ghahramani multiplier problem (2007);
- questions of Dales-Lau-Strauss and Daws (2011/12) on topological invariant means on weakly almost periodic functionals.

### Contributed Talks

#### Speaker: Eman Aldabbas (University of Alberta)

Title: On the weak amenability of the Bench algebra  $\mathcal{SS}(E)$ 

Abstract: The strictly singular operators,  $\mathcal{SS}(X)$ , form a closed ideal in the Banach algebra of all bounded operators on a Banach space X, which contains the ideal of all compact operators  $\mathcal{K}(X)$ . In some cases, the two ideals coincide, but if  $X = L^p[0, 1], p \neq 2$ , or X = C[0, 1], the two ideals are different. However, a classical result of V. Milman shows that on these spaces, the composition of any two strictly singular operators is a compact one. I will show that  $\mathcal{K}(E) \subsetneq \mathcal{SS}(E)$  for any separable  $\mathfrak{L}^p$ -space E which is not isomorphic to  $\ell^p$  (1 . Moreover, I will show that for such Banach spaces, the Banach $algbera <math>\mathcal{SS}(E)$  is not weakly amenable.

#### Speaker: Jason Crann (Carleton University)

### Title: Operator amenability of $A_{cb}(G)$

Abstract: In this talk we examine the connection between weak amenability of a locally compact group G and operator amenability of  $A_{cb}(G)$ , the cb-norm closure of A(G). For QSIN groups, we show that the notions are equivalent, and we give explicit examples of weakly amenable non-QSIN groups for which  $A_{cb}(G)$  is not operator amenable. This talk is based on joint work with Zsolt Tanko.

### Speaker: Hamidreza Farhadi (University of Manitoba)

Title: A new proof for the existence of Haar measure

Abstract: The purpose of this talk is to demonstrate a new method for the construction of Haar measure on an arbitrary locally compact group. In this method we try to reduce the general case (locally compact groups) to smaller classes of groups in which Haar measure can be constructed more easily.

### Speaker: Joel Feinstein (University of Nottingham, UK)

### Title: A general method for constructing essential uniform algebras

Abstract: This is joint work with Alex Izzo, Bowling Green State University. The work was supported by the London Mathematical Society (LMS Scheme 2, grant ref 21212).

We present a general method for constructing essential uniform algebras with prescribed properties. Using this method, we are able to construct the following examples: an essential, natural, regular uniform algebra on the closed unit disc; an essential, natural counterexample to the peak point conjecture on each manifold of dimension at least three; and an essential, natural uniform algebra on the unit sphere in  $\mathbb{C}^3$  containing the ball algebra and invariant under the action of the 3-torus. These examples show that a smoothness hypothesis in some results of Anderson and Izzo cannot be omitted.

### Mahya Ghandehari (University of Delaware)

Title: On character stability of semilattice  $\ell^1$ -algebras

Abstract: Let  $\mathcal{A}$  be a Banach algebra. A bounded linear functional  $\phi$  on  $\mathcal{A}$  is said to be approximately multiplicative if

$$\sup\{|\phi(xy) - \phi(x)\phi(y)|: x, y \in \mathcal{A}, ||x||, ||y|| \le 1\}$$
(1)

is small. For example if  $\phi$  is multiplicative then the quantity defined in (1) is zero. In this talk, we consider approximately multiplicative functionals on weighted convolution algebras of semilattices, and study whether all such functionals arise as small perturbations of multiplicative functionals. A Banach algebra with such a property is said to have stable characters. For a semilattice S and a weight function  $w : S \to [1, \infty)$ , we give intrinsic conditions which answer the question whether  $\ell_w^1(S)$  has stable characters. Our main result states that if S is a semilattice with "infinite breadth" then one can construct a weight w on S such that  $\ell_w^1(S)$  does not have stable characters.

This talk is based on a joint work with Yemon Choi and Hung Le Pham.

#### Speaker: Prachi Loliencar (University of Alberta)

#### Title: Density of Means on Semigroup Flows

Abstract: Hindman and Strauss [2006, 2009] introduced upper and lower densities on semigroups, generalizing the existing concepts on the natural numbers. We will introduce these concepts on semigroup flows and give some results that display their properties. Amenability plays a key role in ensuring the expected properties of the densities on the natural numbers extend to the semigroup and semigroup flows cases.

#### Speaker: Davood Malekzadeh Varnosfaderani (University of Manitoba)

#### Title: Topological center of certain Banach algebras related to locally compact groups

Abstract: We introduce new Banach algebras  $M(G)^*_{\sigma 0}$  and  $L^{\infty}(G)_{\sigma 0}$  and consider their duals endowed with Arens products. We use the strong irregularity of  $L^1(G)$  and M(G)and demonstrate that the toplogical center of  $(M(G)^*_{\sigma 0})^*$  and  $L^{\infty}(G)^*_{\sigma 0}$  are M(G) and  $L^1(G)$  respectively.

#### Speaker: Mehdi Monfared (University of Windsor)

#### Title: Følner's condition and the expansion of Cayley graphs for group actions

Abstract: Graphs with zero expansion are sometimes called amenable graphs (McMullen). Thus a graph is amenable if and only if there exists a sequence  $(F_n)_n$  of finite sets of vertices such that  $\lim_{n\to\infty} |b(F_n)|/|F_n| = 0$  (where  $|\cdot|$  denotes cardinality and  $b(F_n)$  denotes the 'border' of  $F_n$ ). Such a sequence is named a Følner's sequence of the graph. In this talk we show that the expansion of a graph is determined by its components. Furthermore, if G is a finitely generated group acting on a set X, then the amenability of the Cayley graph Cay(G, X) is independent of a particular generating set. Next we show that if G is a group acting on a set X, then the amenability of the pair (G, X) (à la Rosenblatt) can be characterized in terms of the amenability of Cay(H, X), where H is a finitely generated subgroup of G. If G is finitely generated, the latter result can be further strengthened by showing that (G, X) and the graph Cay(G, X) have the same Følner's sequences. This implies that the geometrical configuration of Cay(G, X) can be of help in constructing joint Følner's sequence for both  $\mathbb{Z}^n$  and  $Cay(\mathbb{Z}^n)$ .

The above extends some known results in the literature for the particular case that X = G, and G acts on itself by group multiplication.

Speaker: **Christopher Ramsey** (University of Manitoba) Title: *Takai duality for non-selfadjoint crossed products*  Abstract: We will give the definition of the crossed product of an operator algebra by a locally compact group acting by completely isometric automorphisms. To this context we can extend the Takai duality theorem which describes the result of taking iterated crossed products by a locally compact abelian group and then its dual group. From this we will obtain applications to the study of semisimplicity for operator algebras. This is joint work with E. Katsoulis.

#### Speaker: Varvara Shepelska (University of Saskatchewan) Title: Weak amenability of weighted Orlicz algebras

Abstract: Orlicz spaces are known as a natural generalization of  $L^p$ -spaces. The study of their algebraic properties was initiated recently in [1], [2], and [3]. It was proved there that under certain assumptions weighted Orlicz spaces over locally compact abelian groups become Banach algebras, and the natural questions regarding symmetry, Arens regularity, and amenability of those algebras were considered. In this talk I am going to present the results on weak amenability of weighted Orlicz algebras obtained with E. Samei and S. Öztop. In particular, for a large family of Young functions, we showed that the necessary and sufficient condition for weak amenability of a weighted Orlicz algebra.

### References

- A. Osançliol and S. Öztop, Weighted Orlicz algebras on locally compact groups, 99 (2015), 399-414
- [2] S. Öztop and E. Samei, Twisted Orlicz algebras, I, Studia Math. 236 (2017), no. 3, 271-296.
- [3] S. Öztop and E. Samei, *Twisted Orlicz algebras, II*, submitted.
- Y. Zhang, Weak amenability of commutative Beurling algebras. Proc. Amer. Math. Soc. 142 (2014), no. 5, 1649-1661.

#### Speaker: **Nico Spronk** (University of Waterloo) Title: On convoluters on $L^p$

Abstract: Some work of Matt Daws's and mine has caught some attention recently. It is encapsulated in two theorems. First, if a locally compact G has approximation property in the sense of Haagerup and Kraus, then every convolution operator on  $L^p(G)$  (1is a pseudo-measure. Many key ideas of this were outlined by Cowling, almost 20 yearsago, but the final proof not given. Second, the double commutant of the algebra ofpseudo-measure is the algebra of convoluters. If <math>p = 2, we gain an elementary proof that the commutant of the von Neumann algebra generated by the left regular representation is the von Neumann algebra generated by the right regular representation; a proof in a similar vein was recently presented by le Pham.

#### Speaker: Ross Stokke (University of Winnipeg)

Title: Completely isometric representations of Arens product algebras

Abstract: For a (completely contractive) Banach algebra A and a (completely) contractive (operator) A-module action Q on X, we introduce the (operator) Fourier space  $(\mathcal{F}_Q(A^*), \|\cdot\|_Q)$  and prove that with respect to an Arens-type product  $\Box$ ,  $\mathcal{F}_Q(A^*)^*$  is the unique (operator) "homogeneous left dual Banach algebra (HLDBA) over A" – the definition of a HLDBA over A will be provided and discussed – for which there is a weak\*-continuous (completely) isometric representation as (completely) bounded operators on  $X^*$  extending the dual module action associated with Q. When  $A = L^1(G)$ , examples of  $\mathcal{F}_Q(A^*)$  include A(G) and LUC(G). By applying our theory to these and other examples, we provide new characterizations of some familiar HLDBAs over A and we recover – and sometimes extend – some known (completely) isometric representation theorems concerning these HLDBAs over A.

### Speaker: Keith Taylor (Dalhousie University)

Title: Projections in  $L^1(G)$ 

Abstract: We provide a survey, including recent results on what is know about self-adjoint idempotents in  $L^1(G)$ , where G is a locally compact group. We will draw connections to the Fourier algebra and the foundations of wavelet analysis.

#### Matthew Wiersma (University of Alberta)

Title: Weak containment by restrictions of induced representations

Abstract: A locally compact group G is a QSIN group if  $L^1(G)$  admits a quasi-central approximate identity. Examples of QSIN groups include every discrete group and every amenable group. The presenter has shown that if H is a closed subgroup of a QSIN group G, then the restriction of the induced representation  $\operatorname{Ind}_H^G \pi$  to H weakly contains  $\pi$  for every unitary representation  $\pi$  of H. Applications to the local properties of the group C\*-algebras C\*(G) and C<sup>\*</sup><sub>r</sub>(G), and to the (weak) "extendability" of Fourier and Fourier-Stieltjes spaces  $A_{\pi}$  and  $B_{\pi}$  will be discussed.

Speaker: **Safoura Zadeh** (Laboratoire Mathématiques de Besançon, Universite de Franche-Comté)

Title: On Isometric Isomorphisms of Weighted  $L^p$ -algebras

Abstract: Wendel proved that if G and H are locally compact topological groups and  $T: L^1(G) \to L^1(H)$  is an isometric isomorphism of the group algebras  $L^1(G)$  and  $L^1(H)$ , then the topological groups G and H are isomorphic. Later, Parrot and independently Strichartz, extended Wendel's result to convolution  $L^p$ -algebras. In this talk, I provide a characterization for isometric isomorphisms between convolution weighted  $L^p$ -algebras,  $1 \le p < \infty, p \ne 2$ , when defined. This is a joint work with Yulia Kuznetsova.