UNIVERSITY OF MANITOBA

DATE: October 24, 2005 | MIDTERM

TITLE PAGE

DEPARTMENT & COURSE NO: 136.130 | TIME: 1 hour

EXAMINATION: Vector Geometry & Linear Algebra | EXAMINER: various

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NAME: (Print in ink) ______________________________

STUDENT NUMBER: ______________

SIGNATURE: (in ink) ______________________________

(I understand that cheating is a serious offense)

Please indicate your instructor and section by checking the appropriate box below:

☐ L01 G. I. Moghaddam M, W, F 9:30am - 10:20am
☐ L02 J. Arino T, Th 8:30am - 9:45am
☐ L03 G. I. Moghaddam M, W, F 1:30pm - 2:20pm
☐ L04 N. Zorboska T, Th 11:30am - 12:45pm

INSTRUCTIONS TO STUDENTS:

This is a 1 hour exam. **Please show your work clearly.**

No texts, notes, calculators, cell phones or other aids are permitted.

This exam has a title page, 7 pages of questions and also 1 blank page for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staples.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 60 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may

Question	Points	Score
1	8	
2	9	
3	10	
4	9	
5	6	
6	10	
7	8	
Total:	60	

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[8] 1. State whether each of the following statements is **true** or **false** .

(a) A system whose augmented matrix is $\left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 5k & k-3 \end{array}\right]$ has a unique solution for all values of k.

(a) ____________

(b) A homogeneous system of 4 equations with 6 variables has infinitely many solutions.

(b) ____________

(c) The product of two elementary matrices is also an elementary matrix.

(c) ____________

(d) For two matrices A, B, if $AB = 0$, then $A = 0$ or $B = 0$.

(d) ____________

(e) Every square matrix is invertible.

(e) ____________

(f) If $\det(A) = 0$, then the homogeneous system $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

(f) ____________

(g) Every matrix has a unique reduced row-echelon form.

(g) ____________

(h) If A, B are upper-triangular matrices, then AB is upper-triangular.

(h) ____________

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[9] 2. Consider the linear system of equation

$$\begin{array}{rrrrl} x_1 & & +2x_3 & +x_4 & = 10 \\ -x_1 & +x_2 & +x_3 & -x_4 & = -5 \\ x_1 & +x_2 & +5x_3 & +2x_4 & = 21 \\ & x_2 & +3x_3 & & = 5 \end{array}$$

(a) Find the reduced row-echelon form of the augmented matrix. Clearly describe your row operations using proper notation.

(b) **Using part (a)**, find all solutions of the above system.

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[10] 3. Let

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 4 \\ -1 & 0 \\ 2 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

Evaluate each of the following expressions or explain why it is not defined.

(a) $AC^T - 2D$.

(b) $CD + AB$.

(c) $\det(2A) + \det(B^3)$.

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[9] 4. Let

$$A = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$$

(a) Find a sequence E_1, E_2 of elementary matrices such that $E_2 E_1 A = I$.

(b) Compute E_1^{-1}, E_2^{-1}.

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[6] 5. Let

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ -6 & 5 & 2 & 7 \\ -1 & 4 & 0 & 0 \\ 2 & 5 & 0 & 1 \end{bmatrix}$$

(a) Compute $\det(A)$.

(b) Is A^T invertible? (Why?).

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[10] 6. Let

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & k \\ 0 & 1 & 1 \end{bmatrix}$$

(a) For which values of k does A have an inverse?

(b) If $k = 2$ and $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$, find A^{-1}.

(c) **Use part (b)** to solve the linear system

$$\begin{aligned} x \quad -y \quad +z &= 3 \\ -2y \quad +2z &= 4 \end{aligned}$$

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[8] 7. (a) Let $A = \begin{bmatrix} 2 & 0 & 3 \\ 4 & 1 & 5 \\ 3 & -1 & 7 \end{bmatrix}$. Find the cofactors C_{23} and C_{22} of A.

(b) If $B^{-1} = \begin{bmatrix} \frac{1}{4} & -1 \\ \frac{1}{8} & \frac{1}{2} \end{bmatrix}$, first find $\det(B)$, and then find $\text{adj}(B)$.