ANSWERS TO EXERCISES

Exercise Set 1.1 (page 6)

1. (a), (c), (f)

3. (a) $x = \frac{3}{7} + \frac{5}{7}t$, $y = t$

   (b) $x_1 = \frac{5}{3}s - \frac{4}{3}t + \frac{7}{3}$, $x_2 = s$, $x_3 = t$ $\quad$ $x_1 = \frac{1}{4}r - \frac{5}{8}s + \frac{3}{4}t - \frac{1}{8}$, $x_2 = r$, $x_3 = s$, $x_4 = t$ $\quad$ $v = \frac{8}{3}q - \frac{2}{3}r + \frac{1}{3}s - \frac{4}{3}t$, $w = q$, $x = r$, $y = s$, $z = t$

4. (a) $\begin{bmatrix} 3 & -2 & -1 \\ 4 & 5 & 3 \\ 7 & 3 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & 2 & 1 \\ 3 & -1 & 4 & 7 \\ 6 & 1 & -1 & 0 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 2 & 0 & -1 & 1 & 1 \\ 0 & 3 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 7 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

5. (a) $\begin{aligned} 2x_1 &= 0 \\ 3x_1 - 4x_2 &= 0 \\ x_2 &= 1 \end{aligned}$ (b) $\begin{aligned} 3x_1 - 2x_3 &= 5 \\ 7x_1 + x_2 + 4x_3 &= -3 \\ -2x_2 + x_3 &= 7 \end{aligned}$

   (c) $\begin{aligned} 7x_1 + 2x_2 + x_3 - 3x_4 &= 5 \\ x_1 + 2x_2 + 4x_3 &= 1 \end{aligned}$ (d) $\begin{aligned} x_1 &= 7 \\ x_2 &= -2 \\ x_3 &= 3 \\ x_4 &= 4 \end{aligned}$

6. (a) $x - 2y = 5$ (b) Let $x = t$; then $t - 2y = 5$. Solving for y yields $y = \frac{1}{2}t - \frac{5}{2}$.

12. (a) The lines have no common point of intersection.
    (b) The lines intersect in exactly one point. (c) The three lines coincide.

Exercise Set 1.2 (page 19)

1. (a), (b), (c), (d), (h), (i), (j)

3. (a) Both (b) Neither (c) Both
   (d) Row-echelon (e) Neither (f) Both

4. (a) $x_1 = -3,\ x_2 = 0,\ x_3 = 7$
   (b) $x_1 = 7t + 8,\ x_2 = -3t + 2,\ x_3 = -t - 5,\ x_4 = t$
   (c) $x_1 = 6s - 3t - 2,\ x_2 = s,\ x_3 = -4t + 7,\ x_4 = -5t + 8,\ x_5 = t$
   (d) Inconsistent

6. (a) $x_1 = 3,\ x_2 = 1,\ x_3 = 2$ (b) $x_1 = -\frac{1}{7} - \frac{3}{7}t,\ x_2 = \frac{1}{7} - \frac{4}{7},\ x_3 = t$
   (c) $x = t - 1,\ y = 2s,\ z = s,\ w = t$ (d) Inconsistent

8. (a) Inconsistent (b) $x_1 = -4,\ x_2 = 2,\ x_3 = 7$
   (c) $x_1 = 3 + 2t,\ x_2 = t$ (d) $x = \frac{8}{5} - \frac{3}{5}t - \frac{3}{5}s,\ y = \frac{1}{10} + \frac{2}{5}t - \frac{1}{10}s,\ z = t,\ w = s$

12. (a), (c), (d)

13. (a) $x_1 = 0,\ x_2 = 0,\ x_3 = 0$ (b) $x_1 = -s,\ x_2 = -t - s,\ x_3 = 4s,\ x_4 = t$
    (c) $w = t,\ x = -t,\ y = t,\ z = 0$

14. **(a)** Only the trivial solution **(b)** $u = 7s - 5t,\ v = -6s + 4t,\ w = 2s,\ x = 2t$
(c) Only the trivial solution

15. **(a)** $I_1 = -1,\ I_2 = 0,\ I_3 = 1,\ I_4 = 2$
(b) $Z_1 = -s - t,\ Z_2 = s,\ Z_3 = -t,\ Z_4 = 0,\ Z_5 = t$

19. $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ are possible answers. **20.** $\alpha = \pi/2,\ \beta = \pi,\ \gamma = 0$

23. If $\lambda = 1$, then $x_1 = x_2 = -\frac{1}{2}s,\ x_3 = s$
If $\lambda = 2$, then $x_1 = -\frac{1}{2}s,\ x_2 = 0,\ x_3 = s$

24. $x = -13/7,\ y = 91/54,\ z = -91/8$ **25.** $a = 1,\ b = -6,\ c = 2,\ d = 10$

30. **(a)** Three lines, at least two of which are distinct **(b)** Three identical lines

32. **(a)** False **(b)** False **(c)** False **(d)** False

Exercise Set 1.3 (page 34)

1. **(a)** Undefined **(b)** 4×2 **(c)** Undefined **(d)** Undefined
(e) 5×5 **(f)** 5×2 **(g)** Undefined **(h)** 5×2

2. $a = 5,\ b = -3,\ c = 4,\ d = 1$

4. **(a)** $\begin{bmatrix} 7 & 2 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ **(b)** $\begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$ **(c)** $\begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$

(d) Undefined **(e)** $\begin{bmatrix} -\frac{1}{4} & \frac{3}{2} \\ \frac{9}{4} & 0 \\ \frac{3}{4} & \frac{9}{4} \end{bmatrix}$ **(f)** $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(g) $\begin{bmatrix} 9 & 1 & -1 \\ -13 & 2 & -4 \\ 0 & 1 & -6 \end{bmatrix}$ **(h)** $\begin{bmatrix} 9 & -13 & 0 \\ 1 & 2 & 1 \\ -1 & -4 & -6 \end{bmatrix}$

5. **(a)** $\begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix}$ **(b)** Undefined **(c)** $\begin{bmatrix} 42 & 108 & 75 \\ 12 & -3 & 21 \\ 36 & 78 & 63 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}$ **(e)** $\begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}$ **(f)** $\begin{bmatrix} 21 & 17 \\ 17 & 35 \end{bmatrix}$

(g) $\begin{bmatrix} 0 & -2 & 11 \\ 12 & 1 & 8 \end{bmatrix}$ **(h)** $\begin{bmatrix} 12 & 6 & 9 \\ 48 & -20 & 14 \\ 24 & 8 & 16 \end{bmatrix}$ **(i)** 61 **(j)** 35 **(k)** (28)

8. **(a)** $\begin{bmatrix} 67 \\ 64 \\ 63 \end{bmatrix} = 6\begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} + 0\begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix} + 7\begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix}$

$\begin{bmatrix} 41 \\ 21 \\ 67 \end{bmatrix} = -2\begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} + 1\begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix} + 7\begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix}$

$\begin{bmatrix} 41 \\ 59 \\ 57 \end{bmatrix} = 4\begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} + 3\begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix} + 5\begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix}$

(b) $\begin{bmatrix} 6 \\ 6 \\ 63 \end{bmatrix} = 3\begin{bmatrix} 6 \\ 0 \\ 7 \end{bmatrix} + 6\begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} + 0\begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$

$\begin{bmatrix} -6 \\ 17 \\ 41 \end{bmatrix} = -2\begin{bmatrix} 6 \\ 0 \\ 7 \end{bmatrix} + 5\begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} + 4\begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$

$\begin{bmatrix} 70 \\ 31 \\ 122 \end{bmatrix} = 7\begin{bmatrix} 6 \\ 0 \\ 7 \end{bmatrix} + 4\begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} + 9\begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$

13. **(a)** $A = \begin{bmatrix} 2 & -3 & 5 \\ 9 & -1 & 1 \\ 1 & 5 & 4 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 4 & 0 & -3 & 1 \\ 5 & 1 & 0 & -8 \\ 2 & -5 & 9 & -1 \\ 0 & 3 & -1 & 7 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}$

16. **(a)** $\begin{bmatrix} -3 & -15 & -11 \\ 21 & -15 & 44 \end{bmatrix}$ **(b)** $\begin{bmatrix} 4 & -7 & -19 & -43 \\ 2 & 2 & 18 & 17 \\ 0 & 5 & 25 & 35 \\ 2 & 3 & 23 & 24 \end{bmatrix}$ **(c)** $\begin{bmatrix} 3 & 3 \\ -1 & 4 \\ 1 & 5 \\ 4 & -4 \\ 0 & 14 \end{bmatrix}$

17. **(a)** A_{11} is a 2×3 matrix and B_{11} is a 2×2 matrix. $A_{11}B_{11}$ does not exist.

(b) $\begin{bmatrix} -1 & 23 & -10 \\ 37 & -13 & 8 \\ 29 & 23 & 41 \end{bmatrix}$

21. **(a)** $\begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix}$ **(b)** $\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & 0 & a_{33} & a_{34} & a_{35} & a_{36} \\ 0 & 0 & 0 & a_{44} & a_{45} & a_{46} \\ 0 & 0 & 0 & 0 & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix}$

(c) $\begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & 0 \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix}$ **(d)** $\begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} & 0 \\ 0 & 0 & 0 & a_{54} & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & a_{65} & a_{66} \end{bmatrix}$

27. One; namely, $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

30. **(a)** Yes; for example, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ **(b)** Yes; for example, $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

32. **(a)** True **(b)** False; for example, $A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$ **(c)** True **(d)** True

Exercise Set 1.4 (page 48)

4. $A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{3}{20} \\ -\frac{1}{5} & \frac{1}{10} \end{bmatrix}, \quad C^{-1} = \begin{bmatrix} -\frac{1}{2} & -2 \\ 1 & 3 \end{bmatrix}, \quad D^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$

7. **(a)** $A = \begin{bmatrix} \frac{5}{13} & \frac{1}{13} \\ -\frac{3}{13} & \frac{2}{13} \end{bmatrix}$ **(b)** $A = \begin{bmatrix} \frac{2}{7} & 1 \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$

(c) $A = \begin{bmatrix} -\frac{2}{5} & 1 \\ -\frac{1}{5} & \frac{3}{5} \end{bmatrix}$ **(d)** $A = \begin{bmatrix} -\frac{9}{13} & \frac{1}{13} \\ \frac{2}{13} & -\frac{6}{13} \end{bmatrix}$

9. (a) $p(A) = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$ (b) $p(A) = \begin{bmatrix} 20 & 7 \\ 14 & 6 \end{bmatrix}$ (c) $p(A) = \begin{bmatrix} 39 & 13 \\ 26 & 13 \end{bmatrix}$

11. $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ **13.** $A^{-1} = \begin{bmatrix} \frac{1}{a_{11}} & 0 & \cdots & 0 \\ 0 & \frac{1}{a_{22}} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \frac{1}{a_{nn}} \end{bmatrix}$ **18.** $C = -A^{-1}BA^{-1}$

19. (a) $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ -1 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

20. (a) One example is $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 5 \end{bmatrix}$. (b) One example is $\begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$.

22. Yes **23.** $A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$ **33.** $\begin{bmatrix} \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & \pm 1 \end{bmatrix}$

34. (a) If A is invertible, then A^T is invertible. (b) True

Exercise Set 1.5 (page 57)

1. (a), (c), (d), (f)

3. (a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

6. (a) $\begin{bmatrix} -7 & 4 \\ 2 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -\frac{5}{39} & \frac{2}{13} \\ \frac{4}{39} & \frac{1}{13} \end{bmatrix}$ (c) Not invertible

8. (a) $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & -1 \\ -2 & 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{\sqrt{2}}{26} & \frac{-3\sqrt{2}}{26} & 0 \\ \frac{4\sqrt{2}}{26} & \frac{\sqrt{2}}{26} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & -\frac{1}{7} & \frac{1}{7} \end{bmatrix}$

(d) Not invertible (e) $\begin{bmatrix} -\frac{4}{5} & \frac{3}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{3}{2} & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ \frac{4}{5} & \frac{2}{5} & -\frac{1}{5} & -\frac{1}{5} \end{bmatrix}$

10. (a) $E_1 = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ (b) $A^{-1} = E_2E_1$ (c) $A = E_1^{-1}E_2^{-1}$

11. (a) $\begin{bmatrix} 1 & -4 & 7 \\ 4 & 5 & -3 \\ 2 & -1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -1 & 0 \\ \frac{4}{3} & \frac{5}{3} & -1 \\ 1 & -4 & 7 \end{bmatrix}$ (c) $\begin{bmatrix} 10 & 9 & -6 \\ 4 & 5 & -3 \\ 1 & -4 & 7 \end{bmatrix}$

14. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 3 & 8 \\ 0 & 1 & 7 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

19. **(b)** Add −1 times the first row to the second row.
Add −1 times the first row to the third row.
Add −1 times the second row to the first row.
Add the second row to the third row.

24. In general, no. Try $b = 1,\ a = c = d = 0$.

Exercise Set 1.6 (page 66)

1. $x_1 = 3,\ x_2 = -1$ **4.** $x_1 = 1,\ x_2 = -11,\ x_3 = 16$

6. $w = -6,\ x = 1,\ y = 10,\ z = -7$

9. **(a)** $x_1 = \frac{16}{3},\ x_2 = -\frac{4}{3},\ x_3 = -\frac{11}{3}$ **(b)** $x_1 = -\frac{5}{3},\ x_2 = \frac{5}{3},\ x_3 = \frac{10}{3}$
(c) $x_1 = 3,\ x_2 = 0,\ x_3 = -4$

11. **(a)** $x_1 = \frac{22}{17},\ x_2 = \frac{1}{17}$ **(b)** $x_1 = \frac{21}{17},\ x_2 = \frac{11}{17}$

13. **(a)** $x_1 = \frac{7}{15},\ x_2 = \frac{4}{15}$ **(b)** $x_1 = \frac{34}{15},\ x_2 = \frac{28}{15}$
(c) $x_1 = \frac{19}{15},\ x_2 = \frac{13}{15}$ **(d)** $x_1 = -\frac{1}{5},\ x_2 = \frac{3}{5}$

15. **(a)** $x_1 = -12 - 3t,\ x_2 = -5 - t,\ x_3 = t$ **(b)** $x_1 = 7 - 3t,\ x_2 = 3 - t,\ x_3 = t$

19. $b_1 = b_3 + b_4,\ b_2 = 2b_3 + b_4$ **21.** $X = \begin{bmatrix} 11 & 12 & -3 & 27 & 26 \\ -6 & -8 & 1 & -18 & -17 \\ -15 & -21 & 9 & -38 & -35 \end{bmatrix}$

22. **(a)** Only the trivial solution $x_1 = x_2 = x_3 = x_4 = 0$; invertible
(b) Infinitely many solutions; not invertible

28. **(a)** $I - A$ is invertible. **(b)** $\mathbf{x} = (I - A)^{-1}\mathbf{b}$

30. Yes, for nonsquare matrices

Exercise Set 1.7 (page 73)

1. **(a)** $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{5} \end{bmatrix}$ **(b)** Not invertible **(c)** $\begin{bmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 3 \end{bmatrix}$

3. **(a)** $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix},\quad A^{-2} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{4} \end{bmatrix},\quad A^{-k} = \begin{bmatrix} 1 & 0 \\ 0 & 1/(-2)^k \end{bmatrix}$

(b) $A^2 = \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{9} & 0 \\ 0 & 0 & \frac{1}{16} \end{bmatrix},\quad A^{-2} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 16 \end{bmatrix},\quad A^{-k} = \begin{bmatrix} 2^k & 0 & 0 \\ 0 & 3^k & 0 \\ 0 & 0 & 4^k \end{bmatrix}$

5. (a) **7.** $a = 2,\ b = -1$

10. **(a)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ **(b)** $\begin{bmatrix} \pm\frac{1}{3} & 0 & 0 \\ 0 & \pm\frac{1}{2} & 0 \\ 0 & 0 & \pm 1 \end{bmatrix}$

11. **(a)** $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ **(b)** No

16. **(b)** Yes **17.** Yes

19. $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix},\ \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix},\ \begin{bmatrix} 4 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix},\ \begin{bmatrix} -1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix},$
$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix},\ \begin{bmatrix} -1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix},\ \begin{bmatrix} 4 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix},\ \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

20. **(a)** Yes **(b)** No (unless $n = 1$) **(c)** Yes **(d)** No (unless $n = 1$)

24. **(a)** $x_1 = \frac{7}{4},\ x_2 = 1,\ x_3 = -\frac{1}{2}$ **(b)** $x_1 = -8,\ x_2 = -4,\ x_3 = 3$

25. $A = \begin{bmatrix} 1 & 10 \\ 0 & -2 \end{bmatrix}$ 26. $\frac{n}{2}(1+n)$

Supplementary Exercises (page 76)

1. $x' = \frac{3}{5}x + \frac{4}{5}y,\ y' = -\frac{4}{5}x + \frac{3}{5}y$
3. One possible answer is
$x_1 - 2x_2 - x_3 - x_4 = 0$
$x_1 + 5x_2 + 2x_4 = 0$
5. $x = 4,\ y = 2,\ z = 3$
7. (a) $a \neq 0,\ b \neq 2$ (b) $a \neq 0,\ b = 2$ (c) $a = 0,\ b = 2$ (d) $a = 0,\ b \neq 2$
9. $K = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$
11. (a) $X = \begin{bmatrix} -1 & 3 & -1 \\ 6 & 0 & 1 \end{bmatrix}$ (b) $X = \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$ (c) $X = \begin{bmatrix} -\frac{113}{37} & -\frac{160}{37} \\ -\frac{20}{37} & -\frac{46}{37} \end{bmatrix}$
13. mpn multiplications and $mp(n-1)$ additions 15. $a = 1,\ b = -2,\ c = 3$
16. $a = 1,\ b = -4,\ c = -5$ 26. $A = -\frac{7}{5},\ B = \frac{4}{5},\ C = \frac{3}{5}$
29. (b) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ d & 0 & c^n \end{bmatrix}$, where $d = \begin{cases} \dfrac{a^n - c^n}{a - c} & \text{if } a \neq c \\ na^{n-1} & \text{if } a = c \end{cases}$

Exercise Set 2.1 (page 94)

1. (a) $M_{11} = 29,\ M_{12} = 21,\ M_{13} = 27,\ M_{21} = -11,\ M_{22} = 13,$
$M_{23} = -5,\ M_{31} = -19,\ M_{32} = -19,\ M_{33} = 19$
(b) $C_{11} = 29,\ C_{12} = -21,\ C_{13} = 27,\ C_{21} = 11,\ C_{22} = 13,$
$C_{23} = 5,\ C_{31} = -19,\ C_{32} = 19,\ C_{33} = 19$
3. 152
4. (a) $\text{adj}(A) = \begin{bmatrix} 29 & 11 & -19 \\ -21 & 13 & 19 \\ 27 & 5 & 19 \end{bmatrix}$ (b) $A^{-1} = \begin{bmatrix} \frac{29}{152} & \frac{11}{152} & -\frac{19}{152} \\ -\frac{21}{152} & \frac{13}{152} & \frac{19}{152} \\ \frac{27}{152} & \frac{5}{152} & \frac{19}{152} \end{bmatrix}$
6. -66 8. $k^3 - 8k^2 - 10k + 95$ 11. $A^{-1} = \begin{bmatrix} 3 & -5 & -5 \\ -3 & 4 & 5 \\ 2 & -2 & -3 \end{bmatrix}$
13. $A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{3}{2} & 1 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$ 15. $A^{-1} = \begin{bmatrix} -4 & 3 & 0 & -1 \\ 2 & -1 & 0 & 0 \\ -7 & 0 & -1 & 8 \\ 6 & 0 & 1 & -7 \end{bmatrix}$
16. $x_1 = 1,\ x_2 = 2$ 18. $x = -\frac{144}{55},\ y = -\frac{61}{55},\ z = \frac{46}{11}$
21. Cramer's rule does not apply. 22. $A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
24. $x = 1,\ y = 0,\ z = 2,\ w = 0$ 31. $\det(A) = 10 \times (-108) = -1080$ 34. One

Exercise Set 2.2 (page 101)

2. (a) -30 (b) -2 (c) 0 (d) 0
4. 30 6. -17 8. 39 11. -2
12. (a) -6 (b) 72 (c) -6 (d) 18
16. (a) $\det(A) = -1$ (b) $\det(A) = 1$ 18. $x = 0,\ -1,\ \frac{1}{2}$

Exercise Set 2.3 (page 109)

1. (a) $\det(2A) = -40 = 2^2 \det(A)$ (b) $\det(-2A) = -448 = (-2)^3 \det(A)$
4. (a) Invertible (b) Not invertible (c) Not invertible (d) Not invertible
6. If $x = 0$, the first and third rows are proportional.
   If $x = 2$, the first and second rows are proportional.
12. (a) $k = \dfrac{5 \pm \sqrt{17}}{2}$ (b) $k = -1$
14. (a) $\begin{bmatrix} \lambda-1 & -2 \\ -2 & \lambda-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} \lambda-2 & -3 \\ -4 & \lambda-3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
    (c) $\begin{bmatrix} \lambda-3 & -1 \\ 5 & \lambda+3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
15. (i) $\lambda^2 - 2\lambda - 3 = 0$ (ii) $\lambda = -1,\ \lambda = 3$ (iii) $\begin{bmatrix} -t \\ t \end{bmatrix}, \begin{bmatrix} t \\ t \end{bmatrix}$
    (i) $\lambda^2 - 5\lambda - 6 = 0$ (ii) $\lambda = -1,\ \lambda = 6$ (iii) $\begin{bmatrix} -t \\ t \end{bmatrix}, \begin{bmatrix} \frac{3}{4}t \\ t \end{bmatrix}$
    (i) $\lambda^2 - 4 = 0$ (ii) $\lambda = -2,\ \lambda = 2$ (iii) $\begin{bmatrix} -\frac{t}{5} \\ t \end{bmatrix}, \begin{bmatrix} -t \\ t \end{bmatrix}$
20. No 21. AB is singular.
22. (a) False (b) True (c) False (d) True
23. (a) True (b) True (c) False (d) True

Exercise Set 2.4 (page 117)

1. (a) 5 (b) 9 (c) 6 (d) 10 (e) 0 (f) 2
3. 22 5. 52 7. $a^2 - 5a + 21$ 9. -65 11. -123
13. (a) $\lambda = 1,\ \lambda = -3$ (b) $\lambda = -2,\ \lambda = 3,\ \lambda = 4$ 16. 275
17. (a) $= -120$ (b) $= -120$ 18. $x = \dfrac{3 \pm \sqrt{33}}{4}$ 22. Equals 0 if $n > 1$

Supplementary Exercises (page 118)

1. $x' = \frac{3}{5}x + \frac{4}{5}y,\ y' = -\frac{4}{5}x + \frac{3}{5}y$ 4. 2
5. $\cos\beta = \dfrac{c^2 + a^2 - b^2}{2ac},\ \cos\gamma = \dfrac{a^2 + b^2 - c^2}{2ab}$ 12. $\det(B) = (-1)^{n(n-1)/2}\det(A)$
13. (a) The ith and jth columns will be interchanged.
    (b) The ith column will be divided by c.
    (c) $-c$ times the jth column will be added to the ith column.
15. (a) $\lambda^3 + (-a_{11} - a_{22} - a_{33})\lambda^2$
    $+ (a_{11}a_{22} + a_{11}a_{33} + a_{22}a_{33} - a_{12}a_{21} - a_{13}a_{31} - a_{23}a_{32})\lambda$
    $+ (a_{11}a_{23}a_{32} + a_{12}a_{21}a_{33} + a_{13}a_{22}a_{31} - a_{11}a_{22}a_{33} - a_{12}a_{23}a_{31} - a_{13}a_{21}a_{32})$
18. (a) $\lambda = -5,\ \lambda = 2,\ \lambda = 4;$ $\begin{bmatrix} -2t \\ t \\ t \end{bmatrix}, \begin{bmatrix} 5t \\ t \\ t \end{bmatrix}, \begin{bmatrix} 7t \\ 19t \\ t \end{bmatrix}$ (b) $\lambda = 1;$ $\begin{bmatrix} \frac{1}{2}t \\ -\frac{1}{2}t \\ t \end{bmatrix}$

Exercise Set 3.1 (page 130)

3. (a) $\overrightarrow{P_1P_2} = (-1, -1)$ (b) $\overrightarrow{P_1P_2} = (-7, -2)$ (c) $\overrightarrow{P_1P_2} = (2, 1)$
   (d) $\overrightarrow{P_1P_2} = (a, b)$ (e) $\overrightarrow{P_1P_2} = (-5, 12, -6)$ (f) $\overrightarrow{P_1P_2} = (1, -1, -2)$
   (g) $\overrightarrow{P_1P_2} = (-a, -b, -c)$ (h) $\overrightarrow{P_1P_2} = (a, b, c)$
5. (a) $P(-1, 2, -4)$ is one possible answer. (b) $P(7, -2, -6)$ is one possible answer.
6. (a) $(-2, 1, -4)$ (b) $(-10, 6, 4)$ (c) $(-7, 1, 10)$
   (d) $(80, -20, -80)$ (e) $(132, -24, -72)$ (f) $(-77, 8, 94)$

8. $c_1 = 2,\ c_2 = -1,\ c_3 = 2$ **10.** $c_1 = c_2 = c_3 = 0$

12. **(a)** $x' = 5,\ y' = 8$ **(b)** $x = -1,\ y = 3$

15. $\mathbf{u} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right),\ \mathbf{v} = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right),$

$\mathbf{u} + \mathbf{v} = \left(\frac{\sqrt{3}-1}{2}, \frac{1-\sqrt{3}}{2}\right),\ \mathbf{u} - \mathbf{v} = \left(\frac{\sqrt{3}+1}{2}, \frac{\sqrt{3}+1}{2}\right)$

Exercise Set 3.2 (page 134)

1. **(a)** 5 **(b)** $\sqrt{13}$ **(c)** 5 **(d)** $2\sqrt{3}$ **(e)** $3\sqrt{6}$ **(f)** 6

3. **(a)** $\sqrt{83}$ **(b)** $\sqrt{17} + \sqrt{26}$ **(c)** $4\sqrt{17}$ **(d)** $\sqrt{466}$
(e) $\left(\frac{3}{\sqrt{61}}, \frac{6}{\sqrt{61}}, -\frac{4}{\sqrt{61}}\right)$ **(f)** 1

9. **(b)** $\left(\frac{3}{5}, \frac{4}{5}\right)$ **(c)** $\left(\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}\right)$

10. A sphere of radius 1 centered at (x_0, y_0, z_0)

16. **(a)** $a = c = 0$ **(b)** At least one of a or c is not zero, that is, $a^2 + c^2 > 0$

17. **(a)** The distance from x to the origin is less than 1. **(b)** $\|x - x_0\| > 1$

Exercise Set 3.3 (page 142)

1. **(a)** -11 **(b)** -24 **(c)** 0 **(d)** 0

3. **(a)** Orthogonal **(b)** Obtuse **(c)** Acute **(d)** Obtuse

5. **(a)** $(6, 2)$ **(b)** $\left(-\frac{21}{13}, -\frac{14}{13}\right)$ **(c)** $\left(\frac{55}{13}, 1, -\frac{11}{13}\right)$ **(d)** $\left(\frac{73}{89}, -\frac{12}{89}, -\frac{32}{89}\right)$

8. **(b)** $(3k, 2k)$ for any scalar k **(c)** $\left(\frac{4}{5}, \frac{3}{5}\right), \left(-\frac{4}{5}, -\frac{3}{5}\right)$

11. $\cos\theta_1 = \frac{\sqrt{10}}{10},\ \cos\theta_2 = \frac{3\sqrt{10}}{10},\ \cos\theta_3 = 0$ **13.** $\pm(1/\sqrt{3}, 1/\sqrt{3}, -1/\sqrt{3})$

16. **(a)** $\frac{10}{3}$ **(b)** $-\frac{6}{5}$ **(c)** $\frac{-60+34\sqrt{3}}{33}$ **(d)** $\frac{1}{2}$

20. $\cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$ **21.** **(b)** $\cos\beta = \frac{b}{\|\mathbf{v}\|},\ \cos\gamma = \frac{c}{\|\mathbf{v}\|}$

27. **(a)** The vector $\mathbf{u}$ is dotted with a scalar. **(b)** A scalar is added to the vector $\mathbf{w}$.
(c) Scalars do not have norms. **(d)** The scalar k is dotted with a vector.

29. No; it merely says that $\mathbf{u}$ is orthogonal to $\mathbf{v} - \mathbf{w}$.

30. $\mathbf{r} = (\mathbf{u} \cdot \mathbf{r})\frac{\mathbf{u}}{\|\mathbf{u}\|^2} + (\mathbf{v} \cdot \mathbf{r})\frac{\mathbf{v}}{\|\mathbf{v}\|^2} + (\mathbf{w} \cdot \mathbf{r})\frac{\mathbf{w}}{\|\mathbf{w}\|^2}$ **31.** Theorem of Pythagoras

Exercise Set 3.4 (page 153)

1. **(a)** $(32, -6, -4)$ **(b)** $(-14, -20, -82)$ **(c)** $(27, 40, -42)$
(d) $(0, 176, -264)$ **(e)** $(-44, 55, -22)$ **(f)** $(-8, -3, -8)$

3. **(a)** $\sqrt{59}$ **(b)** $\sqrt{101}$ **(c)** 0

7. For example, $(1, 1, 1) \times (2, -3, 5) = (8, -3, -5)$

9. **(a)** -3 **(b)** 3 **(c)** 3 **(d)** -3 **(e)** -3 **(f)** 0

11. **(a)** No **(b)** Yes **(c)** No **13.** $\left(\frac{6}{\sqrt{61}}, -\frac{3}{\sqrt{61}}, \frac{4}{\sqrt{61}}\right), \left(-\frac{6}{\sqrt{61}}, \frac{3}{\sqrt{61}}, -\frac{4}{\sqrt{61}}\right)$

15. $2(\mathbf{v} \times \mathbf{u})$ **17.** **(a)** $\frac{\sqrt{26}}{2}$ **(b)** $\frac{\sqrt{26}}{3}$ **21.** **(a)** $\sqrt{122}$ **(b)** $\theta \approx 40°19''$

23. **(a)** $\mathbf{m} = (0, 1, 0)$ and $\mathbf{n} = (1, 0, 0)$ **(b)** $(-1, 0, 0)$ **(c)** $(0, 0, -1)$

28. $(-8, 0, -8)$ **31.** **(a)** $\frac{2}{3}$ **(b)** $\frac{1}{2}$ **35.** **(b)** $\mathbf{u} \cdot \mathbf{w} \neq 0,\ \mathbf{v} \cdot \mathbf{w} = 0$

36. No, the equation is equivalent to $\mathbf{u} \times (\mathbf{v} - \mathbf{w}) = \mathbf{0}$ and hence to $\mathbf{v} - \mathbf{w} = k\mathbf{u}$ for some scalar k.

38. They are collinear.

Exercise Set 3.5 (page 162)

1. **(a)** $-2(x + 1) + (y - 3) - (z + 2) = 0$ **(b)** $(x - 1) + 9(y - 1) + 8(z - 4) = 0$
(c) $2z = 0$ **(d)** $x + 2y + 3z = 0$

3. **(a)** $(0, 0, 5)$ is a point in the plane and $\mathbf{n} = (-3, 7, 2)$ is a normal vector so that $-3(x - 0) + 7(y - 0) + 2(z - 5) = 0$ is a point-normal form; other points and normals yield other correct answers.
(b) $(x - 0) + 0(y - 0) - 4(z - 0) = 0$ is a possibility

5. (a) Not parallel (b) Parallel (c) Parallel
9. (a) $x = 3 + 2t,\ y = -1 + t,\ z = 2 + 3t$ (b) $x = -2 + 6t,\ y = 3 - 6t,\ z = -3 - 2t$
(c) $x = 2,\ y = 2 + t,\ z = 6$ (d) $x = t,\ y = -2t,\ z = 3t$
11. (a) $x = -12 - 7t,\ y = -41 - 23t,\ z = t$ (b) $x = \frac{5}{2}t,\ y = 0,\ z = t$
13. (a) Parallel (b) Not parallel 17. $2x + 3y - 5z + 36 = 0$
19. (a) $z - z_0 = 0$ (b) $x - x_0 = 0$ (c) $y - y_0 = 0$ 21. $5x - 2y + z - 34 = 0$
23. $y + 2z - 9 = 0$ 27. $x + 5y + 3z - 18 = 0$
29. $4x + 13y - z - 17 = 0$ 31. $3x - y - z - 2 = 0$
37. (a) $x = \frac{11}{23} + \frac{7}{23}t,\ y = -\frac{41}{23} - \frac{1}{23}t,\ z = t$ (b) $x = -\frac{2}{5}t,\ y = 0,\ z = t$
39. (a) $\frac{5}{3}$ (b) $\frac{1}{\sqrt{29}}$ (c) $\frac{4}{\sqrt{3}}$
43. (a) $\frac{x-3}{2} = y + 1 = \frac{z-2}{3}$ (b) $\frac{x+2}{6} = -\frac{y-3}{6} = -\frac{z+3}{2}$
44. (a) $x - 2y - 17 = 0$ and $x + 4z - 27 = 0$ is one possible answer.
(b) $x - 2y = 0$ and $-7y + 2z = 0$ is one possible answer.
45. (a) $\theta \approx 35°$ (b) $\theta \approx 79°$ 47. They are identical.

Exercise Set 4.1 (page 178)

1. (a) $(-1, 9, -11, 1)$ (b) $(22, 53, -19, 14)$ (c) $(-13, 13, -36, -2)$
(d) $(-90, -114, 60, -36)$ (e) $(-9, -5, -5, -3)$ (f) $(27, 29, -27, 9)$
3. $c_1 = 1,\ c_2 = 1,\ c_3 = -1,\ c_4 = 1$ 5. (a) $\sqrt{29}$ (b) 3 (c) 13 (d) $\sqrt{31}$
8. $k = \pm\frac{5}{7}$ 10. (a) $\left(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right), \left(-\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}\right)$
14. (a) Yes (b) No (c) Yes (d) No (e) No (f) Yes
15. (a) $k = -3$ (b) $k = -2,\ k = -3$ 19. $x_1 = 1,\ x_2 = -1,\ x_3 = 2$
22. The component in the **a** direction is $\text{proj}_{\mathbf{a}}\mathbf{u} = \frac{4}{15}(-1, 1, 2, 3)$; the orthogonal component is $\frac{1}{15}(34, 11, 52, -27)$.
23. They do not intersect.
33. (a) Euclidean measure of "box" in R^n: $a_1a_2\cdots a_n$
(b) Length of diagonal: $\sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}$
35. (a) $d(\mathbf{u}, \mathbf{v}) = \sqrt{2}$
37. (a) True (b) True (c) False (d) True (e) True, unless $\mathbf{u} = \mathbf{0}$

Exercise Set 4.2 (page 193)

1. (a) Linear; $R^3 \to R^2$ (b) Nonlinear; $R^2 \to R^3$ (c) Linear; $R^3 \to R^3$
(d) Nonlinear; $R^4 \to R^2$
3. $\begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}$; $T(-1, 2, 4) = (3, -2, -3)$
5. (a) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 1 & 3 \\ 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 2 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
(d) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{bmatrix}$
7. (a) $T(-1, 4) = (5, 4)$ (b) $T(2, 1, -3) = (0, -2, 0)$

9. **(a)** $(2,-5,-3)$ **(b)** $(2,5,3)$ **(c)** $(-2,-5,3)$

13. **(a)** $\left(-2, \frac{\sqrt{3}-2}{2}, \frac{1+2\sqrt{3}}{2}\right)$ **(b)** $(0,1,2\sqrt{2})$ **(c)** $(-1,-2,2)$

15. **(a)** $\left(-2, \frac{\sqrt{3}+2}{2}, \frac{-1+2\sqrt{3}}{2}\right)$ **(b)** $(-2\sqrt{2},1,0)$ **(c)** $(1,2,2)$

17. **(a)** $\begin{bmatrix} 0 & 0 \\ 1/2 & -\sqrt{3}/2 \end{bmatrix}$ **(b)** $\begin{bmatrix} -\sqrt{2} & \sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix}$ **(c)** $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

19. **(a)** $\begin{bmatrix} \sqrt{3}/8 & -\sqrt{3}/16 & 1/16 \\ 1/8 & 3/16 & -\sqrt{3}/16 \\ 0 & 1/8 & \sqrt{3}/8 \end{bmatrix}$ **(b)** $\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$

21. **(a)** Yes **(b)** No

24. $\begin{bmatrix} \frac{1}{3}(1-\cos\theta)+\cos\theta & \frac{1}{3}(1-\cos\theta)-\frac{1}{\sqrt{3}}\sin\theta & \frac{1}{3}(1-\cos\theta)-\frac{1}{\sqrt{3}}\sin\theta \\ \frac{1}{3}(1-\cos\theta)-\frac{1}{\sqrt{3}}\sin\theta & \frac{1}{3}(1-\cos\theta)+\cos\theta & \frac{1}{3}(1-\cos\theta)-\frac{1}{\sqrt{3}}\sin\theta \\ \frac{1}{3}(1-\cos\theta)-\frac{1}{\sqrt{3}}\sin\theta & \frac{1}{3}(1-\cos\theta)-\frac{1}{\sqrt{3}}\sin\theta & \frac{1}{3}(1-\cos\theta)+\cos\theta \end{bmatrix}$

28. **(c)** 90°

29. **(a)** Twice the orthogonal projection on the x-axis
(b) Twice the reflection about the x-axis

30. **(a)** The x-coordinate is stretched by a factor of 2 and the y-coordinate is stretched by a factor of 3.
(b) Rotation through 30°

31. Rotation through the angle 2θ **34.** Only if $b = 0$.

Exercise Set 4.3 (page 206)

1. **(a)** Not one-to-one **(b)** One-to-one **(c)** One-to-one **(d)** One-to-one
(e) One-to-one **(f)** One-to-one **(g)** One-to-one

3. For example, the vector $(1,3)$ is not in the range.

5. **(a)** One-to-one; $\begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$; $T^{-1}(w_1, w_2) = \left(\frac{1}{3}w_1 - \frac{2}{3}w_2, \frac{1}{3}w_1 + \frac{1}{3}w_2\right)$
(b) Not one-to-one
(c) One-to-one; $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$; $T^{-1}(w_1, w_2) = (-w_2, -w_1)$ **(d)** Not one-to-one

7. **(a)** Reflection about the x-axis **(b)** Rotation through the angle $-\pi/4$
(c) Contraction by a factor of $\frac{1}{3}$ **(d)** Reflection about the yz-plane
(e) Dilation by a factor of 5

9. **(a)** Linear **(b)** Nonlinear **(c)** Linear **(d)** Nonlinear

12. **(a)** For a reflection about the y-axis, $T(\mathbf{e}_1) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ and $T(\mathbf{e}_2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Thus, $T = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.

(b) For a reflection about the xz-plane, $T(\mathbf{e}_1) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $T(\mathbf{e}_2) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$, and

$T(\mathbf{e}_3) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Thus, $T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(c) For an orthogonal projection on the x-axis, $T(\mathbf{e}_1) = \begin{bmatrix}1\\0\end{bmatrix}$ and $T(\mathbf{e}_2) = \begin{bmatrix}0\\0\end{bmatrix}$.

Thus, $T = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$.

(d) For an orthogonal projection on the yz-plane, $T(\mathbf{e}_1) = \begin{bmatrix}0\\0\\0\end{bmatrix}$, $T(\mathbf{e}_2) = \begin{bmatrix}0\\1\\0\end{bmatrix}$, and

$T(\mathbf{e}_3) = \begin{bmatrix}0\\0\\1\end{bmatrix}$. Thus, $T = \begin{bmatrix}0 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$.

(e) For a rotation through a positive angle θ, $T(\mathbf{e}_1) = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}$ and $T(\mathbf{e}_2) = \begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix}$.

Thus, $T = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$.

(f) For a dilation by a factor $k \geq 1$, $T(\mathbf{e}_1) = \begin{bmatrix}k\\0\\0\end{bmatrix}$, $T(\mathbf{e}_2) = \begin{bmatrix}0\\k\\0\end{bmatrix}$, and $T(\mathbf{e}_3) = \begin{bmatrix}0\\0\\k\end{bmatrix}$.

Thus, $T = \begin{bmatrix}k & 0 & 0\\0 & k & 0\\0 & 0 & k\end{bmatrix}$.

13. (a) $T(\mathbf{e}_1) = \begin{bmatrix}-1\\0\end{bmatrix}$ and $T(\mathbf{e}_2) = \begin{bmatrix}0\\0\end{bmatrix}$. Thus, $T = \begin{bmatrix}-1 & 0\\0 & 0\end{bmatrix}$.

(b) $T(\mathbf{e}_1) = \begin{bmatrix}0\\-1\end{bmatrix}$ and $T(\mathbf{e}_2) = \begin{bmatrix}1\\0\end{bmatrix}$. Thus, $T = \begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$.

(c) $T(\mathbf{e}_1) = \begin{bmatrix}0\\3\end{bmatrix}$ and $T(\mathbf{e}_2) = \begin{bmatrix}0\\0\end{bmatrix}$. Thus, $T = \begin{bmatrix}0 & 0\\3 & 0\end{bmatrix}$.

16. (a) Linear transformation from $R^2 \to R^3$; one-to-one
(b) Linear transformation from $R^3 \to R^2$; not one-to-one

17. (a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{3}{4}, \frac{\sqrt{3}}{4}\right)$ (c) $\left(\frac{1-5\sqrt{3}}{4}, \frac{15-\sqrt{3}}{4}\right)$

19. (a) $\lambda = 1$; $\begin{bmatrix}0\\s\\t\end{bmatrix}$ $\lambda = -1$; $\begin{bmatrix}t\\0\\0\end{bmatrix}$ (b) $\lambda = 1$; $\begin{bmatrix}s\\0\\t\end{bmatrix}$ $\lambda = 0$; $\begin{bmatrix}0\\t\\0\end{bmatrix}$

(c) $\lambda = 2$; all vectors in R^3 are eigenvectors (d) $\lambda = 1$; $\begin{bmatrix}0\\0\\t\end{bmatrix}$

23. (a) $\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{bmatrix}$ (b) $\left(\dfrac{1+5\sqrt{3}}{2}, \dfrac{\sqrt{3}-5}{2}\right)$

27. (a) The range of T is a proper subset of R^n.
(b) T must map infinitely many vectors to 0.

Exercise Set 4.4 (page 217)

1. (a) $x^2 + 2x - 1 - 2(3x^2 + 2) = -5x^2 + 2x - 5$ 4. (a) Yes; $A = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix}$

7. (a) $L\colon P_1 \to P_1$ where L maps $ax + b$ to $(a + b)x + a - b$

9. (a) $3e^t + 3e^{-t} = 6\cosh(t)$ (b) Yes

12. $y = 2x^2$ 14. (a) $y = x^3 - x$ 15. (a) $y = 2x^3 - 2x + 2$

18. (a) No, because of the arbitrary constant of integration
(b) No (except for P_0)

21. (a) Each $L_i(x)$ is a polynomial of degree at most n and hence so is the sum $y_0L(x) + \cdots + y_nL(x)$; also, $p(x_i) = 0 + 0 + \cdots + 0 + y_i \cdot L_i(x_i) + 0 + \cdots + 0 + 0 = y_i$, showing that this function is an interpolant of degree at most n.
(b) It is $I_{n+1}\mathbf{c} = \mathbf{y}$ where $\mathbf{c}$ is the vector of c_i values and $\mathbf{y}$ is the vector of y-values.

Exercise Set 5.1 (page 226)

1. Not a vector space. Axiom 8 fails.
3. Not a vector space. Axioms 9 and 10 fail.
5. The set is a vector space under the given operations.
7. The set is a vector space under the given operations.
9. Not a vector space. Axioms 1, 4, 5, and 6 fail.
11. The set is a vector space under the given operations.
13. The set is a vector space under the given operations.
25. No. A vector space must have a zero element.
26. No. Axioms 1, 4, and 6 will fail.
29. (1) Axiom 7 (2) Axiom 4 (3) Axiom 5 (4) Follows from statement 2 (5) Axiom 3 (6) Axiom 5 (7) Axiom 4
32. No; $\mathbf{0}_1 = \mathbf{0}_1 + \mathbf{0}_2 = \mathbf{0}_2$

Exercise Set 5.2 (page 238)

1. (a), (c) 3. (a), (b), (d) 5. (a), (b), (d)

6. (a) Line; $x = -\frac{1}{2}t,\ y = -\frac{3}{2}t,\ z = t$ (b) Line; $x = 2t,\ y = t,\ z = 0$ (c) Origin (d) Origin (e) Line; $x = -3t,\ y = -2t,\ z = t$ (f) Plane; $x - 3y + z = 0$

9. (a) $-9 - 7x - 15x^2 = -2\mathbf{p}_1 + \mathbf{p}_2 - 2\mathbf{p}_3$ (b) $6 + 11x + 6x^2 = 4\mathbf{p}_1 - 5\mathbf{p}_2 + \mathbf{p}_3$
(c) $0 = 0\mathbf{p}_1 + 0\mathbf{p}_2 + 0\mathbf{p}_3$ (d) $7 + 8x + 9x^2 = 0\mathbf{p}_1 - 2\mathbf{p}_2 + 3\mathbf{p}_3$

11. (a) The vectors span. (b) The vectors do not span.
(c) The vectors do not span. (d) The vectors span.

12. (a), (c), (e) 15. $y = z$

24. (a) They span a line if they are collinear and not both $\mathbf{0}$. They span a plane if they are not collinear.
(b) If $\mathbf{u} = a\mathbf{v}$ and $\mathbf{v} = b\mathbf{u}$ for some real numbers a, b
(c) We must have $\mathbf{b} = \mathbf{0}$ since a subspace must contain $\mathbf{x} = \mathbf{0}$ and then $\mathbf{b} = A\mathbf{0} = \mathbf{0}$.

26. (a) For example, $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$
(b) The set of matrices having one entry equal to 1 and all other entries equal to 0

Exercise Set 5.3 (page 248)

1. (a) $\mathbf{u}_2$ is a scalar multiple of $\mathbf{u}_1$.
(b) The vectors are linearly dependent by Theorem 5.3.3.
(c) $\mathbf{p}_2$ is a scalar multiple of $\mathbf{p}_1$. (d) B is a scalar multiple of A.

3. None 5. (a) They do not lie in a plane. (b) They do lie in a plane.

7. (b) $\mathbf{v}_1 = \frac{2}{7}\mathbf{v}_2 - \frac{3}{7}\mathbf{v}_3,\ \ \mathbf{v}_2 = \frac{7}{2}\mathbf{v}_1 + \frac{3}{2}\mathbf{v}_3,\ \ \mathbf{v}_3 = -\frac{7}{3}\mathbf{v}_1 + \frac{2}{3}\mathbf{v}_2$ 9. $\lambda = -\frac{1}{2},\ \lambda = 1$

18. If and only if the vector is not zero

19. (a) They are linearly independent since $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ do not lie in the same plane when they are placed with their initial points at the origin.
(b) They are not linearly independent since $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ lie in the same plane when they are placed with their initial points at the origin.

20. (a), (d), (e), (f)

24. (a) False (b) False (c) True (d) False 27. (a) Yes

Exercise Set 5.4 (page 263)

1. **(a)** A basis for R^2 has two linearly independent vectors.
(b) A basis for R^3 has three linearly independent vectors.
(c) A basis for P_2 has three linearly independent vectors.
(d) A basis for M_{22} has four linearly independent vectors.

3. (a), (b) **7.** **(a)** $(\mathbf{w})_S = (3, -7)$ **(b)** $(\mathbf{w})_S = \left(\frac{5}{28}, \frac{3}{14}\right)$ **(c)** $(\mathbf{w})_S = \left(a, \frac{b-a}{2}\right)$

9. **(a)** $(\mathbf{v})_S = (3, -2, 1)$ **(b)** $(\mathbf{v})_S = (-2, 0, 1)$ **11.** $(A)_S = (-1, 1, -1, 3)$

13. Basis: $\left(-\frac{1}{4}, -\frac{1}{4}, 1, 0\right)$, $(0, -1, 0, 1)$; dimension $= 2$

15. Basis: $(3, 1, 0)$, $(-1, 0, 1)$; dimension $= 2$

19. **(a)** 3-dimensional **(b)** 2-dimensional **(c)** 1-dimensional

20. 3-dimensional

21. **(a)** $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{e}_1\}$ or $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{e}_2\}$ **(b)** $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{e}_1\}$ or $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{e}_2\}$ or $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{e}_3\}$

27. **(a)** One possible answer is $\{-1 + x - 2x^2, 3 + 3x + 6x^2, 9\}$.
(b) One possible answer is $\{1 + x, x^2, -2 + 2x^2\}$.
(c) One possible answer is $\{1 + x - 3x^2\}$.

29. **(a)** $(2, 0)$ **(b)** $\left(\frac{2}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$ **(c)** $(0, 1)$ **(d)** $\left(\frac{2}{\sqrt{3}}a, b - \frac{a}{\sqrt{3}}\right)$

31. Yes; for example, $\begin{bmatrix} 1 & 0 \\ 0 & \pm 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ \pm 1 & 0 \end{bmatrix}$

32. **(a)** n **(b)** $n(n+1)/2$ **(c)** $n(n+1)/2$

35. **(a)** The dimension is $n - 1$.
(b) $(1, 0, 0, \ldots, 0, -1)$, $(0, 1, 0, \ldots, 0, -1)$, $(0, 0, 1, \ldots, 0, -1)$, $\ldots$, $(0, 0, 0, \ldots, 1, -1)$ is a basis of size $n - 1$.

Exercise Set 5.5 (page 276)

1. $\mathbf{r}_1 = (2, -1, 0, 1)$, $\mathbf{r}_2 = (3, 5, 7, -1)$, $\mathbf{r}_3 = (1, 4, 2, 7)$;

$$\mathbf{c}_1 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \quad \mathbf{c}_2 = \begin{bmatrix} -1 \\ 5 \\ 4 \end{bmatrix}, \quad \mathbf{c}_3 = \begin{bmatrix} 0 \\ 7 \\ 2 \end{bmatrix}, \quad \mathbf{c}_4 = \begin{bmatrix} 1 \\ -1 \\ 7 \end{bmatrix}$$

3. **(a)** $\begin{bmatrix} -2 \\ 10 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ -6 \end{bmatrix}$ **(b)** $\mathbf{b}$ is not in the column space of A.

(c) $\begin{bmatrix} 1 \\ 9 \\ 1 \end{bmatrix} - 3\begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + (t-1)\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} + t\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

(e) $\begin{bmatrix} 4 \\ 3 \\ 5 \\ 7 \end{bmatrix} = -26\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + 13\begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} - 7\begin{bmatrix} 0 \\ 2 \\ 1 \\ 2 \end{bmatrix} + 4\begin{bmatrix} 1 \\ 1 \\ 3 \\ 2 \end{bmatrix}$

5. **(a)** $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 3 \\ 1 \end{bmatrix}$; $t\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} -2 \\ 7 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$; $t\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix}-1\\0\\0\\0\end{bmatrix}+r\begin{bmatrix}2\\1\\0\\0\end{bmatrix}+s\begin{bmatrix}-1\\0\\1\\0\end{bmatrix}+t\begin{bmatrix}-2\\0\\0\\1\end{bmatrix}$; $r\begin{bmatrix}2\\1\\0\\0\end{bmatrix}+s\begin{bmatrix}-1\\0\\1\\0\end{bmatrix}+t\begin{bmatrix}-2\\0\\0\\1\end{bmatrix}$

(d) $\begin{bmatrix}\frac{6}{5}\\\frac{7}{5}\\0\\0\end{bmatrix}+s\begin{bmatrix}\frac{7}{5}\\\frac{4}{5}\\1\\0\end{bmatrix}+t\begin{bmatrix}\frac{1}{5}\\-\frac{3}{5}\\0\\1\end{bmatrix}$; $s\begin{bmatrix}\frac{7}{5}\\\frac{4}{5}\\1\\0\end{bmatrix}+t\begin{bmatrix}\frac{1}{5}\\-\frac{3}{5}\\0\\1\end{bmatrix}$

7. (a) $\mathbf{r}_1=[1\ \ 0\ \ 2]$, $\mathbf{r}_2=[0\ \ 0\ \ 1]$, $\mathbf{c}=\begin{bmatrix}1\\0\\0\end{bmatrix}$, $\mathbf{c}_2=\begin{bmatrix}2\\1\\0\end{bmatrix}$

(b) $\mathbf{r}_1=[1\ \ -3\ \ 0\ \ 0]$, $\mathbf{r}_2=[0\ \ 1\ \ 0\ \ 0]$, $\mathbf{c}_1=\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$, $\mathbf{c}_2=\begin{bmatrix}-3\\1\\0\\0\end{bmatrix}$

(c) $\mathbf{r}_1=[1\ \ 2\ \ 4\ \ 5]$, $\mathbf{r}_2=[0\ \ 1\ \ -3\ \ 0]$,
$\mathbf{r}_3=[0\ \ 0\ \ 1\ \ -3]$, $\mathbf{r}_4=[0\ \ 0\ \ 0\ \ 1]$,
$\mathbf{c}_1=\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}$, $\mathbf{c}_2=\begin{bmatrix}2\\1\\0\\0\\0\end{bmatrix}$, $\mathbf{c}_3=\begin{bmatrix}4\\-3\\1\\0\\0\end{bmatrix}$, $\mathbf{c}_4=\begin{bmatrix}5\\0\\-3\\1\\0\end{bmatrix}$

(d) $\mathbf{r}_1=[1\ \ 2\ \ -1\ \ 5]$, $\mathbf{r}_2=[0\ \ 1\ \ 4\ \ 3]$,
$\mathbf{r}_3=[0\ \ 0\ \ 1\ \ -7]$, $\mathbf{r}_4=[0\ \ 0\ \ 0\ \ 1]$,
$\mathbf{c}_1=\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$, $\mathbf{c}_2=\begin{bmatrix}2\\1\\0\\0\end{bmatrix}$, $\mathbf{c}_3=\begin{bmatrix}-1\\4\\1\\0\end{bmatrix}$, $\mathbf{c}_4=\begin{bmatrix}5\\3\\-7\\1\end{bmatrix}$

9. (a) $\begin{bmatrix}1\\5\\7\end{bmatrix}$, $\begin{bmatrix}-1\\-4\\-6\end{bmatrix}$ (b) $\begin{bmatrix}2\\4\\0\end{bmatrix}$ (c) $\begin{bmatrix}1\\2\\-1\end{bmatrix}$, $\begin{bmatrix}4\\1\\3\end{bmatrix}$

(d) $\begin{bmatrix}1\\3\\-1\\2\end{bmatrix}$, $\begin{bmatrix}4\\-2\\0\\3\end{bmatrix}$ (e) $\begin{bmatrix}1\\0\\2\\3\\-2\end{bmatrix}$, $\begin{bmatrix}-3\\3\\-3\\-6\\9\end{bmatrix}$, $\begin{bmatrix}2\\6\\-2\\0\\2\end{bmatrix}$

11. (a) $(1, 1, -4, -3)$, $(0, 1, -5, -2)$, $(0, 0, 1, -\frac{1}{2})$
(b) $(1, -1, 2, 0)$, $(0, 1, 0, 0)$, $(0, 0, 1, -\frac{1}{6})$
(c) $(1, 1, 0, 0)$, $(0, 1, 1, 1)$, $(0, 0, 1, 1)$, $(0, 0, 0, 1)$

14. (b) $\begin{bmatrix}0&0&0\\0&1&0\\0&0&1\end{bmatrix}$ 17. $\begin{bmatrix}3a&-5a\\3b&-5b\end{bmatrix}$ for all real numbers a, b not both 0.

Exercise Set 5.6 (page 288)

1. Rank $(A)=\text{rank}(A^T)=2$
3. (a) 2; 1 (b) 1; 2 (c) 2; 2 (d) 2; 3 (e) 3; 2
5. (a) Rank = 4, nullity = 0 (b) Rank = 3, nullity = 2 (c) Rank = 3, nullity = 0

7. (a) Yes, 0 (b) No (c) Yes, 2 (d) Yes, 7 (e) No
(f) Yes, 4 (g) Yes, 0

9. $b_1 = r,\ b_2 = s,\ b_3 = 4s - 3r,\ b_4 = 2r - s,\ b_5 = 8s - 7r$ 11. No

13. Rank is 2 if $r = 2$ and $s = 1$; the rank is never 1.

16. (a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (b) A line through the origin (c) A plane through the origin
(d) The nullspace is a line through the origin and the row space is a plane through the origin.

19. (a) 3 (b) 5 (c) 3 (d) 3

Supplementary Exercises (page 290)

1. (a) All of R^3 (b) Plane: $2x - 3y + z = 0$
(c) Line: $x = 2t,\ y = t,\ z = 0$ (d) The origin: $(0, 0, 0)$

3. (a) $a(4, 1, 1) + b(0, -1, 2)$ (b) $(a + c)(3, -1, 2) + b(1, 4, 1)$
(c) $a(2, 3, 0) + b(-1, 0, 4) + c(4, -1, 1)$

5. (a) $\mathbf{v} = (-1 + r)\mathbf{v}_1 + \left(\frac{2}{3} - r\right)\mathbf{v}_2 + r\mathbf{v}_3$; r arbitrary 7. No

9. (a) Rank $= 2$, nullity $= 1$ (b) Rank $= 3$, nullity $= 2$
(c) Rank $= n + 1$, nullity $= n$

11. $\{1, x^2, x^3, x^4, x^5, x^6, \ldots, x^n\}$ 13. (a) 2 (b) 1 (c) 2 (d) 3

Exercise Set 6.1 (page 304)

1. (a) 2 (b) 11 (c) -13 (d) -8 (e) 0 3. (a) 3 (b) 56

5. (b) 29 7. (a) $\begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{5} \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 \\ 0 & \sqrt{6} \end{bmatrix}$

9. (a) No. Axiom 4 fails. (b) No. Axioms 2 and 3 fail.
(c) Yes (d) No. Axiom 4 fails.

11. (a) $3\sqrt{2}$ (b) $3\sqrt{5}$ (c) $3\sqrt{13}$ 13. (a) $\sqrt{74}$ (b) 0

15. (a) $\sqrt{105}$ (b) $\sqrt{47}$ 17. (a) $\sqrt{2},\ \frac{1}{3}\sqrt{6},\ \frac{1}{5}\sqrt{10}$ (b) $\frac{2}{3}\sqrt{6}$

19. $\langle \mathbf{u}, \mathbf{v} \rangle = \frac{1}{9}u_1v_1 + u_2v_2$

23. No for P_3, since $\mathbf{p} = x\left(x - \frac{1}{2}\right)(x - 1)$ satisfies $\langle \mathbf{p}, \mathbf{p} \rangle = 0$

27. (a) $-\frac{28}{15}$ (b) 0 34. $a = 1/25,\ b = 1/16$

Exercise Set 6.2 (page 315)

1. (a) Yes (b) No (c) Yes (d) No (e) No (f) Yes

5. (a) $-\frac{1}{\sqrt{2}}$ (b) $-\frac{3}{\sqrt{73}}$ (c) 0 (d) $-\frac{20}{9\sqrt{10}}$ (e) $-\frac{1}{\sqrt{2}}$ (f) $\frac{2}{\sqrt{55}}$

9. (a) Orthogonal (b) Orthogonal (c) Orthogonal (d) Not orthogonal

11. $\pm\frac{1}{57}(-34, 44, -6, 11)$

15. (a) $x = t,\ y = -2t,\ z = -3t$ (b) $2x - 5y + 4z = 0$ (c) $x - z = 0$

17. (a) $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}; \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

18. (a) $(16, 19, 1)$ (b) $(0, 1, 0),\ \left(\frac{1}{2}, 0, 1\right)$ (c) $(-1, -1, 1, 0),\ \left(\frac{2}{7}, -\frac{4}{7}, 0, 1\right)$
(d) $(-1, -1, 1, 0, 0),\ (-2, -1, 0, 1, 0),\ (-1, -2, 0, 0, 1)$

32. $\langle \mathbf{u}, \mathbf{v} \rangle = \frac{1}{2}u_1v_1 + \frac{1}{6}u_2v_2$

35. (a) The line $y = -x$ (b) The xz-plane (c) The x-axis

37. (a) False (b) True (c) True (d) False

Exercise Set 6.3 (page 328)

1. (a), (b), (d) 3. (b), (d) 5. (a)