THE UNIVERSITY OF MANITOBA

DATE: December 12, 2008 — **FINAL EXAMINATION**

PAPER NO: 462 — TITLE PAGE

DEPARTMENT & COURSE NO: MATH 1300 — TIME: 2 hours

EXAMINATION: Vector Geom. & Lin. Alg. — EXAMINER: Various

LAST NAME (Family Name): (Print in ink) ______________________________

FIRST NAME (Given Name): (Print in ink) ______________________________

STUDENT NUMBER: ____________________ SEAT NUMBER: ________________

SIGNATURE: (In ink) ____
(I understand that cheating is a serious offense)

Please mark your section number.

- ❑ Section A01 MWF (9:30 – 10:20) M. Doob
- ❑ Section A02 T & R (8:30 – 9:45) G.I. Moghadddam
- ❑ Section A03 T & R (11:30 – 12:45) R. Craigen
- ❑ Section A04 MWF (11:30 – 12:20) N. Zorboska
- ❑ Section A05 MWF (1:30 – 2:20) D. Kelly
- ❑ Section A91 Challenge for Credit SJR
- ❑ Deferred Exam

INSTRUCTIONS TO CANDIDATES:

This is a 2 hour exam. **Please show your work clearly**. Please justify your answers, unless otherwise stated.

No calculators or other aids are permitted.

This exam has a title page, 10 pages of questions and 2 blank pages for rough work. Please check that you have all the pages.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 120.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

DO NOT WRITE IN THIS COLUMN	
1.	/ 13
2.	/ 14
3.	/ 14
4.	/ 13
5.	/ 10
6.	/ 11
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8.	/ 12
9.	/ 12
10.	/ 8
TOTAL	/120

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Values

[13] 1. Let each of the matrices $A = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \end{array}\right]$ and $B = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$ be the augmented matrix of a linear system. Answer the following questions.

(a) How many equations and how many variables does the system with the augmented matrix A have?

(b) Find all solutions, if any, of the system with the augmented matrix A. If there are no solutions, say so.

(c) How many equations and how many variables does the system with the augmented matrix B have?

(d) Find all solutions, if any, of the system with augmented matrix B. If there are no solutions, say so.

Values

[14] 2. Let $A = \begin{bmatrix} 2 & 4 & -2 \\ 2 & k & 4 \\ 0 & 2 & 6 \end{bmatrix}$;

(a) Find the value(s) of k for which the matrix A is **not** invertible.

(b) For $k = 0$ in the matrix A (see above), find all solutions of the linear system $2A^{-1}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

(c) For $k = 1$ in the matrix A (see above), find the cofactors C_{13} and C_{32}.

Values

[14] 3.

(a) Give an example of a nonzero 2 x 2 matrix A such that $A^T = -A$.

(b) Let A be a 2008 x 2008 matrix such that $I + A^2 = 0$, find all possible values for $\det(A)$.

(c) If $A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix}$, find an elementary matrix E such that

$AB = EB$.

Values

[13] 4.

(a) Find the cosine of the angle θ between $\mathbf{u} = (1, 2, -1)$ and $\mathbf{v} = (3, 0, 4)$.

(b) Find the area of the triangle PQR, where $P(1, 2, 3)$, $Q(5, 4, 4)$ and $R(3, 3, 4)$.

(c) Find the volume of the parallelepiped whose sides are the vectors $\mathbf{u} = (1, 2, -1)$, $\mathbf{v} = (3, 0, 4)$ and $\mathbf{w} = (-2, 1, 0)$.

Values

[10] 5.

(a) Find the point of intersection of the line $x = 6 + 2t$, $y = 5 + 4t$, $z = 17 + 5t$ and the plane $2x + 2y + z = 5$.

(b) Find the equation of the plane that is perpendicular to line of 5(a) and passes through the point $(2,1,4)$.

(c) Find the distance from the point $(2,3,-1)$ to the plane $7x + y + 9z = 10$.

Values

[11] 6.

(a) Find a unit vector in the direction of $\mathbf{u} = (5, -2, 1, -3)$.

(b) Find the value of t so that the vector $(1, t, 3, 4t)$ is orthogonal to $(3, 2, 1, 5)$.

(c) Let $P(1,2,0)$ and $Q(-5,0,5)$. Find a linear equation that describes all points $R(x, y, z)$ so that $\|\overrightarrow{PR}\| = \|\overrightarrow{QR}\|$.

Values

[13] 7. Let $A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

(a) Show that the set $\{A, B, C\}$ is linearly independent in M_{22}.

(b) Is the matrix D in the span of $\{A, B, C\}$? Show your work.

(c) Is the set $\{A, B, C\}$ a basis for M_{22} ? Explain.

(d) Is the set $\{A, B, C, D\}$ a basis for M_{22} ? Explain.

Values

[12] 8. In each question below determine if W is a subspace of the vector space V. Show your work.

(a) $V = \mathbb{R}^4$ and W is the set of all triples of the form $t(1,0,-1,2)$ with t in $\mathbb{R}$, ie. $W = \{t(1,0,-1,2) \mid t \text{ in } \mathbb{R}\}$.

(b) $V = M_{22}$ and W is the set of all 2 x 2 matrices $\begin{bmatrix} a & b \\ 0 & -1 \end{bmatrix}$ with a and b in $\mathbb{R}$, ie. $W = \left\{ \begin{bmatrix} a & b \\ 0 & -1 \end{bmatrix} \mid a, b \text{ in } \mathbb{R} \right\}$.

(c) $V = P_3$ and W is the set of all polynomials of the form $a + bx^2$ with $a \geq 0$ and $b \geq 0$, ie. $W = \{p(x) = a + bx^2 \mid a \geq 0, b \geq 0\}$.

Values

[12] 9. $R = \begin{bmatrix} 1 & -1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ is the reduced row echelon form of the matrix

$$A = \begin{bmatrix} 2 & -2 & 1 & -1 & 0 \\ 2 & -1 & 1 & 1 & -2 \\ 0 & 0 & -2 & 6 & 1 \\ 1 & -1 & 0 & -2 & 0 \end{bmatrix}$$

(a) (i) Find a basis for the null space of A .

(ii) The dimension of the null space of A is ________________.

(b) (i) Give a basis for the row space of A .

(ii) The dimension of the row space of A is ________________.

(c) (i) Give a basis for the column space of A .

(ii) The dimension of the column space of A is _____________.

Values

[8] 10. Answer the following questions by filling in the blanks with either true or false, or with the required number.

(a) If A is a 6 x 6 invertible matrix, the dimension of its row space is ____________ .

(b) If $\{\mathbf{u}, \mathbf{v}\}$ is a basis for $\mathbb{R}^2$, then $\mathbf{u}$ must be orthogonal to $\mathbf{v}$. ____________

(c) If A is a 5 x 6 matrix and the dimension of the null space of A is 2, then the dimension of the row space of A is ____________ .

(d) The line $x = -1 + t$, $y = 3t$, $z = -t$, t in $\mathbb{R}$, is a subspace of $\mathbb{R}^3$. ____________

(e) The vectors $\mathbf{u} = (1, -2, -1, 0, 1)$ and $\mathbf{v} = (-2, 4, a, 0, -2)$ are linearly dependant when $a =$ ____________ .

(f) The linear system $x - 2y - z = 1$, $-2x + 4y + 2z = a$ is inconsistent when $a \neq$ ____________ .

(g) If $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ are polynomials in P_2 such that $\mathbf{p}_1(0) = \mathbf{p}_2(0) = \mathbf{p}_3(0) = 0$, then the set $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$ is linearly dependent. ____________

(h) There exists a subspace of $\mathbb{R}^7$ with exactly five vectors. ____________