Surname (Print)________________________________

Given Name(s) (Print)____________________________

I understand that cheating is a serious offense.

Signature:(in ink)______________________________

Student Number________________________________

Room ______________Seat Number______________

THE UNIVERSITY OF MANITOBA
DEPARTMENT OF MATHEMATICS

MATH 1300 Vector Geometry and Linear Algebra

Mid-Term Exam (Deferred)

Date: Tuesday, March 9, 2010
Time: 5:30–6:30 PM

Identify your section by marking an X in the box.

	Section	Instructor	Slot	Time	Room
☐	A01	C. Platt	3	10:30–11:20 am	221 Wallace
☐	A02	N. Zorboska	5	TTh 10:00–11:15am	208 Armes
☐	A03	C. Sun	8	MWF 1:30–2:20pm	204 Armes
☐	A04	D. Kalajdzievska	12	MWF 3:30–4:20 pm	208 Armes
☐	A05	M. Doob	15	TTh 4:00–5:15pm	200 Armes
☐	Other	(challenge, deferred, etc.)			

DO NOT WRITE IN THIS COLUMN

1	/8
2	/8
3	/7
4	/7
5	/8
6	/8
7	/7
8	/7
Total	/60

Instructions

*Fill in **all** the information above.*

This is a one-hour exam.

***No** calculators, texts, notes, or other aids are permitted.*

***Show your work clearly** for full marks.*

*This exam has 8 questions on 4 numbered pages, for a total of 60 points. **Check now** that you have a complete exam.*

*Answer all questions on the exam paper in the space provided. If you need more room, you may continue your answer on the **reverse** side, but **clearly indicate** that your work is continued there. You may also use the backs of pages for scratch work, but none of it will be marked unless clearly indicated otherwise.*

*If a question calls for a specific method, **no credit** will be given for other methods.*

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Tuesday, March 9, 2010, 5:30–6:30 PM | Mid-Term Exam (Deferred)

PAGE NO: 1 of 4

COURSE: MATH 1300 Vector Geometry and Linear Algebra | TIME: 1 HOUR

EXAMINERS:

[Values]

[8] **1.** Use Gaussian elimination to convert the following augmented matrix to row-echelon form, and then find all solutions to the system that it represents:

$$[A|\mathbf{b}] = \left[\begin{array}{cccc|c} -1 & -1 & -8 & 0 & 5 \\ 0 & 6 & -12 & 24 & -6 \\ 1 & 3 & 6 & 1 & -3 \end{array}\right]$$

[8] **2.** Suppose that the augmented matrix of a system $A\mathbf{x} = \mathbf{b}$ has been row-reduced to

$$\left[\begin{array}{cc|c} 1 & 0 & -2+2b \\ 0 & 2a+5 & b \end{array}\right].$$

(a) Find all a and b such that system has no solutions.

(b) Find all a and b such that system has infinitely many solutions.

(c) Find all a and b such that system has exactly one solution. Write the solution for every such a and b.

[Values]

[7] **3.** $A = \begin{bmatrix} 2 & 3 \\ 1 & -2 \\ 7 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 0 & 1 \\ 3 & 1 & 5 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$, $E = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

Given the above, state whether each of the following expressions is well defined, and if so, find the resulting value. If it is not defined, state why.

(a) $DB^T - E$

(b) $B + \frac{2}{3}D^2$

(c) $2CA + B$

(d) The third column (only) of BC

(e) The $(3,1)$ entry of BA

[7] **4.** Suppose A is a 4×4 invertible matrix.

(a) What is the reduced row echelon form of $4A$?

(b) For any 4×1 matrix $\mathbf{b}$, determine the number of solutions of the system $A\mathbf{x} = \mathbf{b}$.

(c) Find $\det(A)$, given that $\det(A^2) + 2\det(A) = 0$. Justify your answer.

[Values]

[8] **5.** State whether each statement is "TRUE" or "FALSE". If it is "FALSE", state a modified version that is true.

(a) For any a, b, and c, $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}^{-1}$ exists and is equal to $\begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$.

(b) If A and B are invertible $n \times n$ matrices, then $(AB)^{-1} = A^{-1}B^{-1}$.

(c) For matrices A, B, and C, if $AB = 0$, then either $A = 0$ or $B = 0$.

(d) The product of any two lower triangular matrices is upper triangular.

[8] **6.** Supppose A and B are 5×5 matrices with $\det(A) = \dfrac{1}{2}$ and $\det(B) = -3$. Compute each of the following:

(a) $\det(2AB)$

(b) $\det((AB^T)^T)$

(c) $\det(BAB^{-1})$

(d) $B\,\mathrm{adj}(B)$

[Values]

[7] **7.** Calculate the following determinants (by any method, answers only will be marked).

(a) $\begin{vmatrix} 2 & 3 & 0 & 1 \\ 1 & 2 & -1 & 1 \\ 0 & 3 & 0 & 4 \\ 0 & 5 & 0 & 2 \end{vmatrix}$

(b) $\begin{vmatrix} 5 & a & 5a & 7 \\ 3 & 0 & 3a & 19 \\ -12 & 14 & -12a & 1 \\ 0 & 33 & 0 & 11a \end{vmatrix}$, where a is any number.

(c) $\begin{vmatrix} -43 & 0 & 0 & 0 & 0 & 0 \\ 20 & 2 & 0 & 0 & 0 & 0 \\ 19 & 13 & -1 & 0 & 0 & 0 \\ 21 & 41 & 0 & 10 & 0 & 0 \\ 7 & 41 & 1 & 5 & 1 & 0 \\ 1 & -1 & 1 & -1 & 1 & -1 \end{vmatrix}$

[7] **8.** Let $A = \begin{bmatrix} 2 & -3 & 3 \\ 1 & 5 & 3 \\ 0 & 0 & 1 \end{bmatrix}$. Then $\det(A) = 13$. (Given, no calculation required).

(a) The cofactor matrix of A is partially given as $\mathrm{cof}(A) = \begin{bmatrix} 5 & -1 & 0 \\ a & b & 0 \\ -24 & -3 & 13 \end{bmatrix}$. Find a and b.

(b) Find the adjoint of A.

(c) Find A^{-1} by the adjoint method (no credit for other methods).