THE UNIVERSITY OF MANITOBA

DATE: December 8, 2006 **FINAL EXAMINATION**

PAPER NO: 88 PAGE 1 of 9

DEPARTMENT & COURSE NO: MATH 1300 TIME: 2 hours

EXAMINATION: Vector Geom. & Lin. Alg. EXAMINER: Various

Values

1. The augmented matrix of a system of linear equations has been reduced to the matrix

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & a(a-1) & a \end{array}\right]$$

[6] (a) Find all of the values of a, if any, for which the system is inconsistent.

[6] (b) Find all of the values of a, if any, for which the system has infinitely many solutions. What is the number of parameters that must be introduced?

[3] (c) Find all values of a, if any, for which the system has unique solution.

Values

[10] 2. **Use Cramer's Rule** to solve for x_3 from the linear system

$$\begin{aligned} -x_1 + 3x_2 - 3x_3 &= 0 \\ 2x_1 + 3x_2 &= 0 \\ 2x_1 - x_2 + x_3 &= 5 \\ 3x_1 - x_2 - 4x_3 + 2x_4 &= 7. \end{aligned}$$

3. Let $A = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 3 & 0 \\ 2 & 1 & 5 & -3 \end{bmatrix}$.

[8] (a) Evaluate the entries a and b in the incomplete adjoint of A:

$$\text{adj}(A) = \begin{bmatrix} -8 & 6 & b & 2 \\ a & 0 & -10 & 0 \\ 5 & 0 & 0 & 0 \\ 8 & 4 & -2 & -2 \end{bmatrix}$$

[2] (b) If you know that $\det(A) = 10$, find A^{-1} by using Part (a).

Values

4. Let $A(1, 0, 1)$, $B(1, 2, 3)$ and $C(3, 2, 1)$ be points in $\mathbb{R}^3$.

[6] (a) Find the area of the triangle with the vertices *A, B* and *C*.

[4] (b) Find $\text{proj}_{\overrightarrow{AB}}\left(\overrightarrow{AC}\right)$.

Values

[10] 5. Given that $\begin{vmatrix} a & b & c \\ d & e & f \\ 1 & -2 & 3 \end{vmatrix} = 5$, find $\begin{vmatrix} a-2 & b+4 & c-6 \\ 2a+3d & 2b+3e & 2c+3f \\ 3 & -6 & 9 \end{vmatrix}$.

Values

6. Let Π be the plane $2x-3z+12=0$, and let $P(-1,1,-1)$ and $Q(1,0,1)$ be two points in $\mathbb{R}^3$.

[6] (a) Find parametric equations of the line ℓ that is perpendicular to the plane Π and that contains the point P.

[5] (b) Find the point of intersection of the line ℓ (from (a)) and the plane Π.

[4] (c) Find the distance between the point P and the plane Π.

Values

7. Let $\mathbf{u} = (1, 3, 0, -2)$ and $\mathbf{v} = (3, -1, 1, -6)$.

[5] (a) Is the set of vectors $\{\mathbf{u}, \mathbf{v}\}$ linearly independent or not? Justify your answer.

[2] (b) Are $\mathbf{u}$ and $\mathbf{v}$ orthogonal or not? Justify your answer.

[4] (c) Find all values of k such that $k\mathbf{u}$ is a unit vector.

Values

8. Let $A=\begin{bmatrix}1 & 1\\ 0 & -2\end{bmatrix}$, $B=\begin{bmatrix}2 & -1\\ 0 & 0\end{bmatrix}$, $C=\begin{bmatrix}0 & 0\\ -1 & -1\end{bmatrix}$.

[5] (a) Determine whether the set $\{A,B,C\}$ is linearly independent or not.

[5] (b) Does $D=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ belong to the span of $\{A,B,C\}$? Explain.

[2] (c) Does $D=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ belong to the span of $\{A,B,C\}$? Explain.

[2] (d) Is the set $\{A,B,C\}$ a basis for M_{22}? Explain.

Values

9. In each question below determine if W is a subspace of the vector space V:

[3] (a) $V = \mathbb{R}^3$ and $W = \{(x,y,z) : x - 2y + z + 3 = 0\}$.

[6] (b) $V = \mathbb{R}^3$ and $W = \{(2t,-t,0) : t \text{ in } \mathbb{R}\}$.

[6] (c) $V = \mathbb{P}_2$ and $W = \{p(x) = ax + 3ax^2 : a \text{ in } \mathbb{R}\}$.

Values

10. Let A be a 5x5 matrix

[4] (a) If A is invertible, find a basis and the dimension of the row space and of the null space of A.

[6] (b) If A is such that its reduced row echelon form

$$R = \begin{bmatrix} 1 & -1 & 0 & 3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find a basis for the row space and a basis for the null space of A. What is the dimension of the column space of A?