

Values

- [10] 1. Let $\mathbf{u} = (-1, 0, 1)$ and let $\mathbf{v} = (-1, -2, 1)$. Calculate, or explain why it is not possible to find:

a) $\|2\mathbf{u} - 3\mathbf{v}\|$,

$$2\mathbf{u} - 3\mathbf{v} = (-2, 0, 2) - (-3, -6, 3) = (1, 6, -1)$$

$$\|2\mathbf{u} - 3\mathbf{v}\| = \sqrt{1 + 6^2 + (-1)^2} = \sqrt{1 + 36 + 1} = \sqrt{38}$$

b) $(\mathbf{u} \cdot \mathbf{v}) \times \mathbf{u}$,

NOT POSSIBLE, $\mathbf{u} \cdot \mathbf{v}$ IS A SCALAR AND NOT A VECTOR IN \mathbb{R}^3 .

c) $\mathbf{u} \cdot \mathbf{v} + \sqrt{3} \cos \theta$, where θ is the angle between \mathbf{u} and \mathbf{v} ,

$$\mathbf{u} \cdot \mathbf{v} = 1 + 0 + 1 = 2, \quad \cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} =$$

$$= \frac{2}{\sqrt{1+1} \sqrt{1+4+1}} = \frac{2}{\sqrt{2} \sqrt{6}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\mathbf{u} \cdot \mathbf{v} + \sqrt{3} \cos \theta = 2 + \sqrt{3} \cdot \frac{1}{\sqrt{3}} = 2 + 1 = 3$$


Values

[10] 2. Let A (1, 1, 2) and B (-2, 1, 0) be two points.

a) Find the orthogonal projection of \vec{AB} onto the vector $\mathbf{a} = (1, 1, -1)$.

$$\vec{AB} = (-3, 0, -2) \quad \text{proj}_{\mathbf{a}} \vec{AB} = \frac{\vec{AB} \cdot \mathbf{a}}{\|\mathbf{a}\|^2} \mathbf{a} = \frac{-3+0+2}{1+1+1} (1, 1, -1) = -\frac{1}{3} (1, 1, -1)$$

b) If the point C has coordinates (k, 1, 2) and the area of the triangle ABC is 3, find k.



$$A_{\Delta} = \frac{1}{2} \|\vec{AB} \times \vec{AC}\|, \quad \vec{AC} = (k-1, 0, 0)$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} + & - & + \\ -3 & 0 & -2 \\ k-1 & 0 & 0 \end{vmatrix} = (0, -(-2)(k-1), 0) = (0, 2(k-1), 0)$$

$$A_{\Delta} = \frac{1}{2} \sqrt{4(k-1)^2} = \frac{1}{2} \cdot 2|k-1| = |k-1| = 3, \quad k=4 \text{ or } k=-2$$

[12] 3. Given the line l with parametric equations $x = 2 - 6t$, $y = 4t$, $z = 1 + 2t$ and the planes $\Pi_1: x + 3y - 3z + 7 = 0$ and $\Pi_2: x + y - z = 0$

a) Show that the line l is parallel to the plane Π_1 .

$$\ell \parallel (-6, 4, 2), \quad \Pi_1 \perp (1, 3, -3)$$

$$(-6, 4, 2) \cdot (1, 3, -3) = -6 + 12 - 6 = 0 \quad \text{AND SO } \ell \parallel \Pi_1$$

b) Find the distance between the line l and the plane Π_1 .

$$P(2, 0, 1), \quad D = \frac{|2 \cdot 1 + 0 \cdot 3 + 1 \cdot (-3) + 7|}{\sqrt{1+9+9}} = \frac{6}{\sqrt{19}}$$

c) Find a point normal equation of the plane through the point P (1, 2, -4) that is perpendicular to both planes Π_1 and Π_2 .

$$\Pi_1 \perp (1, 3, -3), \quad \Pi_2 \perp (1, 1, -1)$$

$$\underline{n} = (1, 3, -3) \times (1, 1, -1) = \begin{vmatrix} + & - & + \\ 1 & 3 & -3 \\ 1 & 1 & -1 \end{vmatrix} = (-3+3, -(-1+3), 1-3)$$

$$\underline{n} = (0, -2, -2) = -2(0, 1, 1)$$

$$0(x-1) + 1(y-2) + 1(z+4) = 0$$

$$y+z+2=0$$

THE UNIVERSITY OF MANITOBA

DATE: February 26, 2004

MIDTERM EXAMINATION

DEPARTMENT & COURSE NO: 136.130

PAGE 3 of 5

EXAMINATION: Vector Geometry & Linear Algebra

TIME: 1 hour

EXAMINERS: Various

Values

[10] 4. Use **Gauss-Jordan elimination** to solve the following system:

$$\begin{aligned} x_1 - 3x_2 + 3x_3 + 6x_4 &= 9 \\ x_1 - 3x_2 + 3x_3 + 7x_4 &= 11 \\ -x_3 - 2x_4 &= 1 \end{aligned}$$

$$\begin{bmatrix} 1 & -3 & 3 & 6 & 9 \\ 1 & -3 & 3 & 7 & 11 \\ 0 & 0 & -1 & -2 & 1 \end{bmatrix} \xrightarrow{R_2 \rightarrow -R_1 + R_2} \begin{bmatrix} 1 & -3 & 3 & 6 & 9 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & -1 & -2 & 1 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3}$$

$$\begin{bmatrix} 1 & -3 & 3 & 6 & 9 \\ 0 & 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{\textcircled{1} -R_2} \begin{bmatrix} 1 & -3 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_1 \rightarrow R_1 - 3R_2}$$

$$\begin{bmatrix} 1 & -3 & 0 & 0 & 12 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 - 2R_3} \begin{bmatrix} \textcircled{1} & -3 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & \textcircled{1} & 2 \end{bmatrix}$$

$$x_2 = t, \quad x_3 = -5, \quad x_1 = 3t + 12, \quad x_4 = 2$$

$$S = \left\{ (3t + 12, t, -5, 2) \mid -\infty < t < \infty \right\}$$

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PAGE 4 of 5

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[7] 5. Given that each of the matrices

$$A = \begin{bmatrix} 1 & 2 & 0 & -3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & -3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

is the reduced row-echelon form of an augmented matrix of a linear system

a) Indicate the number of solutions of each of the corresponding systems by placing an X in the following table:

	A	B	C
Unique solution		X	
No solution	X		
Infinitely many solutions			X

b) For each of the systems that has solutions, find the solution set.

$$B : \quad x_1 = 4, \quad x_2 = -2$$

$$C : \quad x_2 = t, \quad x_4 = s$$

$$x_3 = 6 - s$$

$$x_1 = 3t$$

$$S = \{ (3t, t, 6-s, s) : -\infty < t < \infty, -\infty < s < \infty \}$$

EXAMINATION: Vector Geometry & Linear Algebra TIME: 1 hour

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[11] 6. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 5 \\ -2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}$ and $C = [4 \ 1 \ 3]$, compute each of the following expressions, or show why it is not defined:

a) $A(B^T - 2B)$,

$$(3 \times 2)(2 \times 2 - 2 \times 2) = (3 \times 2)(2 \times 2) = 3 \times 2$$

$$B^T - 2B = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 2 \\ -4 & 8 \end{bmatrix} = \begin{bmatrix} -3 & -4 \\ 5 & -4 \end{bmatrix}$$

$$A(B^T - 2B) = \begin{bmatrix} 1 & 1 \\ 0 & 5 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} -3 & -4 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} 2 & -8 \\ 25 & -20 \\ -9 & 20 \end{bmatrix}$$

b) $CA + 3B^T$,

$$(1 \times 3)(3 \times 2) + (2 \times 2) = (1 \times 2) + (2 \times 2) \text{ NOT POSSIBLE}$$

c) $(CA)B$,

$$((1 \times 3)(3 \times 2))(2 \times 2) = (1 \times 2)(2 \times 2) = 1 \times 2$$

$$CA = [4 \ 1 \ 3] \begin{bmatrix} 1 & 1 \\ 0 & 5 \\ -2 & -3 \end{bmatrix} = [-2 \ 0]$$

$$[-2 \ 0] \begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix} = [-6 \ -2]$$

d) $BA^T + CA$.

$$(2 \times 2)(2 \times 3) + (1 \times 3)(3 \times 2) = (2 \times 3) + (1 \times 2)$$

NOT POSSIBLE