EXAMINATION: Vector Geometry & Linear Algebra TIME: 1 hour

EXAMINERS: Various

Values

[10] 1. Let $\mathbf{u} = (-1, 0, 1)$ and let $\mathbf{v} = (-1, -2, 1)$. Calculate, or explain why it is not possible to find:

a) $\| 2\mathbf{u} - 3\mathbf{v} \|$,

$2\underline{u} - 3\underline{v} = (-2, 0, 2) - (-3, -6, 3) = (1, 6, -1)$

$\| 2\underline{u} - 3\underline{v} \| = \sqrt{1 + 6^2 + (-2)^2} = \sqrt{1+36+4} = \sqrt{38}$

b) $(\mathbf{u} \cdot \mathbf{v})$ x $\mathbf{u}$,

NOT POSSIBLE, $\underline{u} \cdot \underline{v}$ IS A SCALAR AND NOT A VECTOR IN $\mathbb{R}^3$

c) $\mathbf{u} \cdot \mathbf{v} + \sqrt{3} \cos\theta$, where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$,

$\underline{u} \cdot \underline{v} = 1 + 0 + 1 = 2$, $\cos\theta = \dfrac{\underline{u} \cdot \underline{v}}{\|\underline{u}\| \, \|\underline{v}\|} =$

$= \dfrac{2}{\sqrt{1+1} \, \sqrt{1+4+1}} = \dfrac{2}{\sqrt{2} \, \sqrt{6}} = \dfrac{2}{2\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\underline{u} \cdot \underline{v} + \sqrt{3} \cos\theta = 2 + \sqrt{3} \cdot \dfrac{1}{\sqrt{3}} = 2 + 1 = 3$

Values

[10] 4. Use **Gauss-Jordan elimination** to solve the following system:

$$
\begin{aligned}
x_1 - 3x_2 + 3x_3 + 6x_4 &= 9 \\
x_1 - 3x_2 + 3x_3 + 7x_4 &= 11 \\
-x_3 - 2x_4 &= 1
\end{aligned}
$$

$$\begin{bmatrix} 1 & -3 & 3 & 6 & 9 \\ 1 & -3 & 3 & 7 & 11 \\ 0 & 0 & -1 & -2 & 1 \end{bmatrix} \xrightarrow{R_2 \to -R_1 + R_2} \begin{bmatrix} 1 & -3 & 3 & 6 & 9 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & -1 & -2 & 1 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3}$$

$$\begin{bmatrix} 1 & -3 & 3 & 6 & 9 \\ 0 & 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 1 & -3 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_1 \to R_1 - 3R_2}$$

$$\begin{bmatrix} 1 & -3 & 0 & 0 & 12 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 2R_3} \begin{bmatrix} 1 & -3 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$x_2 = t$, $x_3 = -5$, $x_1 = 3t + 12$, $x_4 = 2$

$S = \{ (3t+12, t, -5, 2) \mid -\infty < t < \infty \}$

Values

[10] 2. Let A (1, 1, 2) and B (–2, 1, 0) be two points.

a) Find the orthogonal projection of $\overrightarrow{AB}$ onto the vector $\mathbf{a}$ = (1, 1, –1).

$$\overrightarrow{AB} = (-3, 0, -2) \qquad \text{PROJ}_{\underline{a}}\ \overrightarrow{AB} = \frac{\overrightarrow{AB}\cdot \underline{a}}{\|\underline{a}\|^2}\,\underline{a} = \frac{-3+0+2}{1+1+1}(1,1,-1) =$$

$$= -\frac{1}{3}(1,1,-1)$$

b) If the point C has coordinates (k, 1, 2) and the area of the triangle ABC is 3, find k .

$$A_\Delta = \frac{1}{2}\,\|\overrightarrow{AB}\times\overrightarrow{AC}\|, \quad \overrightarrow{AC} = (k-1, 0, 0)$$

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{array}{ccc} + & - & + \\ -3 & 0 & -2 \\ k-1 & 0 & 0 \end{array} = (0, -(-2)(k-1), 0) = (0, 2(k-1), 0)$$

$$A_\Delta = \frac{1}{2}\sqrt{4(k-1)^2} = \frac{1}{2}\,2\,|k-1| = |k-1| = 3, \quad k = 4 \text{ OR } k = -2$$

[12] 3. Given the line l with parametric equations x = 2 – 6t , y = 4t , z = 1 + 2t and the planes Π_1: x + 3y – 3z + 7 = 0 and Π_2: x + y – z = 0

a) Show that the line l is parallel to the plane Π_1 .

$$l \parallel (-6, 4, 2), \quad \Pi_1 \perp (1, 3, -3)$$

$$(-6,4,2)\cdot(1,3,-3) = -6+12-6 = 0 \quad \text{AND SO } l \parallel \Pi_1$$

b) Find the distance between the line l and the plane Π_1.

$$P(2,0,1), \quad D = \frac{|2\cdot 1 + 0\cdot 3 + 1(-3) + 7|}{\sqrt{1+9+9}} = \frac{6}{\sqrt{19}}$$

c) Find a point normal equation of the plane through the point P (1, 2, –4) that is perpendicular to both planes Π_1 and Π_2.

$$\Pi_1 \perp (1,3,-3), \quad \Pi_2 \perp (1,1,-1)$$

$$\underline{n} = (1,3,-3)\times(1,1,-1) = \begin{array}{ccc} + & - & + \\ 1 & 3 & -3 \\ 1 & 1 & -1 \end{array} = (-3+3, -(-1+3), 1-3)$$

$$\underline{n} = (0,-2,-2) = -2(0,1,1)$$

$$0(x-1) + 1(y-2) + 1(z+4) = 0$$

$$y + z + 2 = 0$$