

THE UNIVERSITY OF MANITOBA

DATE: February 26, 2004

MIDTERM EXAMINATION

DEPARTMENT & COURSE NO: 136.130

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EXAMINATION: Vector Geometry & Linear Algebra

TIME: 1 hour

EXAMINERS: Various

Values

[7] 5. Given that each of the matrices

$$A = \begin{bmatrix} 1 & 2 & 0 & -3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & -3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

is the reduced row-echelon form of an augmented matrix of a linear system

a) Indicate the number of solutions of each of the corresponding systems by placing an X in the following table:

	A	B	C
Unique solution		X	
No solution	X		
Infinitely many solutions			X

b) For each of the systems that has solutions, find the solution set.

B : $x_1 = 4, \quad x_2 = -2$

C : $x_2 = t, \quad x_4 = b$

$x_3 = 6 - s$

$x_1 = 3t$

$$S = \left\{ (3t, t, 6-s, b) : -\infty < t < \infty, -\infty < s < \infty \right\}$$

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[11] 6. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 5 \\ -2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}$ and $C = [4 \ 1 \ 3]$, compute each of the following expressions, or show why it is not defined:

a) $A(B^T - 2B)$,
 $(3 \times 2)(2 \times 2 - 2 \times 2) = (3 \times 2)(2 \times 2) = 3 \times 2$

$$B^T - 2B = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 2 \\ -4 & 8 \end{bmatrix} = \begin{bmatrix} -3 & -4 \\ 5 & -4 \end{bmatrix}$$

$$A(B^T - 2B) = \begin{bmatrix} 1 & 1 \\ 0 & 5 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} -3 & -4 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} 2 & -8 \\ 25 & -20 \\ -9 & 20 \end{bmatrix}$$

b) $CA + 3B^T$,

$$(1 \times 3)(3 \times 2) + (2 \times 2) = (1 \times 2) + (2 \times 2) \text{ NOT POSSIBLE}$$

c) $(CA)B$,

$$((1 \times 3)(3 \times 2))(2 \times 2) = (1 \times 2)(2 \times 2) = 1 \times 2$$

$$CA = [4 \ 1 \ 3] \begin{bmatrix} 1 & 1 \\ 0 & 5 \\ -2 & -3 \end{bmatrix} = [-2 \ 0]$$

$$[-2 \ 0] \begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix} = [-6 \ -2]$$

d) $BA^T + CA$.

$$(2 \times 2)(2 \times 3) + (1 \times 3)(3 \times 2) = (2 \times 3) + (1 \times 2)$$

NOT POSSIBLE