THE UNIVERSITY OF MANITOBA

DATE: April 14, 2003 — FINAL EXAMINATION

PAPER NO: 131 —

DEPARTMENT & COURSE NO: 136.130 — TIME: 2 HOURS

EXAMINATION: Vector Geometry & Linear Algebra — EXAMINERS: Various

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Values

[10] 1. Let $A=\begin{pmatrix}1&0&2\\2&1&1\\1&1&2\end{pmatrix}$, $B=\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}$, $C=\begin{pmatrix}1&3\\-2&-1\\1&2\end{pmatrix}$.

Either evaluate each of the following expressions or give a reason why it is not defined.

(a) $2B-3C^T$
(b) CAB
(c) $A(C+B)$
(d) $\operatorname{tr}(BC)-\operatorname{tr}(CB)$

[9] 2. Find all the values of b for which the following system of equations:

$$\begin{aligned} x+2y&=b\\ 2x+\left(3+b^2\right)y&=3b+1\end{aligned}$$

(a) has no solutions;
(b) has infinitely many solutions;
(c) has a unique solution.

[10] 3. Express the matrix $M=\begin{pmatrix}6&-2\\-5&2\end{pmatrix}$ as a product of elementary matrices.

[12] 4. Let A be the coefficient matrix of the following system of equations.

$$\begin{aligned} 2x+6y-2z&=1\\ y-z&=2\\ x+4y-z&=3\end{aligned}$$

(a) Write the system in the form of a matrix equation.
(b) Find the cofactor C_{21} for the matrix A.
(c) Find A^{-1}.
(d) Solve the system using part 4(d) above.

[12] 5. Let $M=\begin{pmatrix}2&a&-1\\1&b&3\\0&c&2\end{pmatrix}$. Although the entries of the second column are unknown, we are given that $|M|=12$.

(a) Solve the system $M\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2\\-2\\0\end{pmatrix}$ FOR y ONLY, using Cramer's rule.

(b) Find the value of the determinant (using the given information about (a))

$$\begin{vmatrix}2c+a&2&3\\2c&0&4\\3a+b&7&0\end{vmatrix}.$$

(c) Find the adjoint of $A=\begin{pmatrix}a&b\\c&d\end{pmatrix}$

Values

[20] 6. Let $\mathbf{u}=(1,1,1)$, $\mathbf{v}=(-1,-2,3)$ and $\mathbf{w}=(2,2,0)$. Find:

(a) $\|2u-3v\|$.
(b) the area of the triangle with vertices at $A(1,1,1,)$, $B(-1,-2,3)$, $C(2,2,0)$.
(c) the volume of the parallelepiped, determined by **u**, **v** and **w**.
(d) show that two of **u**, **v** and **w** are orthogonal.
(e) find the projection of **w** onto **u**.

[8] 7. Consider the vectors **u**, **v**, where $\|\mathbf{u}\|=2$, $\|\mathbf{v}\|=3$, and the angle between **u** and **v** is $\theta=\frac{\pi}{3}$ radians $\left(=60^\circ\right)$. Find

(a) $\mathbf{u}\cdot\mathbf{v}$.
(b) $\|\mathbf{u}\times\mathbf{v}\|$.

[12] 8. Let P be the plane whose equation is $x+3y-z=2$. Define the points $A(5,0,3)$, $B(2,-1,3)$, $C(1,1,2)$ and $D(0,0,1)$.

(a) Find a vector normal to the plane.
(b) Show that A and C are on the plane P but B and D are not.
(c) Find parametric equations for the line orthogonal to P and passing through A.

[10] 9. Consider the vector space $P_2=\{a+bx+cx^2 \mid a,b,c\in\mathbf{R}\}$ and the subsets $S=\{f(x)\in P_2 \mid f(2)=0\}$ and $T=\{a+bx+cx^2\in P_2 \mid a,b,c\geq -1\}$.

(a) Use the subspace test to show that S is a subspace of P_2.
(b) Show that T is not a subspace of P_2, by giving a specific case for which part of the subspace test fails.

[5] 10. Define “linearly independent set”.

[12] 11. Let $A=\begin{pmatrix}1&2&3&1\\1&2&3&2\\2&4&6&0\\1&2&2&1\end{pmatrix}$.

(a) Find a basis for, and dimension of, the row space of A.
(b) Find a basis for, and dimension of, the column space of A.
(c) Find a basis for, and dimension of, the solution space of the system $A\mathbf{x}=\mathbf{0}$.