THE UNIVERSITY OF MANITOBA

DATE: April 18, 2005 FINAL EXAMINATION

PAPER NO: 300 TITLE PAGE

DEPARTMENT & COURSE NO: 136.130 TIME:__2_HOURS

EXAMINATION: Vector Geometry & Linear Algebra EXAMINERS: Various

---

LAST NAME: (Print in ink) ____

GIVEN NAME: (Print in ink) ____

STUDENT NUMBER: ________________________

SEAT NUMBER ____________________________

SIGNATURE: (in ink) ____
(I understand that cheating is a serious offense)

Please mark your section number.

SECTION		TIME	INSTRUCTOR
❑ L05	Tues., Thurs.	10:00 – 11:20	**V. Charette**
❑ L06	M,W,F	1:30 – 2:20	**N. Zorboska**
❑ L07	M,W,F	1:30 – 2:20	**K. Doerksen**
❑ L08	M,W,F	2:30 – 3:20	**R. Thomas**
❑ L09	Tues. eve	7:00 – 9:45	**J. Sichler**
❑ L92	**Challenge for credit**		
❑ Deferral			

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly**.
Please justify your answers unless otherwise stated.

No calculators or cell phones are permitted.

This exam has a title page, 9 pages of questions and 2 blank pages for rough work. Please check that you have all the pages.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 120.

Answer all questions on the exam paper in the space provided

THE UNIVERSITY OF MANITOBA

DATE: April 18, 2005 FINAL EXAMINATION

PAPER NO: 300 PAGE 2 of 8

DEPARTMENT & COURSE #: 136.130 TIME: 2 HOURS

EXAMINATION: Vector Geom. & Linear Algebra EXAMINERS: Various

---

[11] 1. The augmented matrix of a system of linear equations has been reduced to the matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & a-1 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & a^2-2a & a-2 \end{array}\right]$$

(a) Determine the values of a such that the system has:

(i) No solutions

(ii) Infinitely many solutions

(iii) A unique solution

THE UNIVERSITY OF MANITOBA

DATE: April 18, 2005 FINAL EXAMINATION

PAPER NO: 300 PAGE 3 of 8

DEPARTMENT & COURSE #: 136.130 TIME: 2 HOURS

EXAMINATION: Vector Geom. & Linear Algebra EXAMINERS: Various

---

[16] 2. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & -1 & 1 \\ 2 & k+1 & k \end{bmatrix}$

(a) For which values of k does A fail to be invertible?

(b) Find A^{-1} for $k = 1$.

(c) **Use A^{-1} from (b)** to solve the system

$$\begin{aligned} x + 2y + z &= -1 \\ -x - y + z &= 3 \\ 2x + 2y + z &= -3 \end{aligned}$$

THE UNIVERSITY OF MANITOBA

DATE: April 18, 2005 FINAL EXAMINATION

PAPER NO: 300 PAGE 4 of 8

DEPARTMENT & COURSE #: 136.130 TIME: 2 HOURS

EXAMINATION: Vector Geom. & Linear Algebra EXAMINERS: Various

---

[8] 3. Use cofactor expansion along column two to calculate the determinant

$$\begin{vmatrix} 1 & -1 & 5 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & -3 & 2 \\ -2 & 0 & 0 & 1 \end{vmatrix}$$

(**No marks will be given** for any other method.)

[8] 4. Let A and B be 4×4 matrices such that $\det(A) = 3$ and $\det(B) = 2$. Calculate:

(a) $\det\left(AB^2\right)$

(b) $\det\left(2A^T B^{-1}\right)$

THE UNIVERSITY OF MANITOBA

DATE: April 18, 2005	FINAL EXAMINATION
PAPER NO: 300	PAGE 5 of 8
DEPARTMENT & COURSE #: 136.130	TIME: 2 HOURS
EXAMINATION: Vector Geom. & Linear Algebra	EXAMINERS: Various

[8] 5. Use Cramer's Rule to solve for y only:

$$\begin{array}{rcrcrcl} x & - & 2y & & & = & 0 \\ & & y & + & 3z & = & 3\,. \\ x & - & y & + & 2z & = & 1 \end{array}$$

(**No marks will be given** for any other method.)

[16] 6. Given the lines $L_1 : (x, y, z) = (5, 4, 3) + s\,(2, 1\,0)$ and $L_2 : (x, y, z) = (0, 2, 4) + t\,(1, 0\,-1)$

(a) Find their point of intersection P.

(b) Find parametric equations of the line L through P from (a), perpendicular to both L_1 and L_2.

THE UNIVERSITY OF MANITOBA

DATE: April 18, 2005 FINAL EXAMINATION

PAPER NO: 300 PAGE 6 of 8

DEPARTMENT & COURSE #: 136.130 TIME: 2 HOURS

EXAMINATION: Vector Geom. & Linear Algebra EXAMINERS: Various

---

[13] 7. Let $\mathbf{a}$ and $\mathbf{b}$ be two nonzero vectors in the 3-space R^3 and let $\mathbf{v} = 2\mathbf{a} - 3\mathbf{b}$.

(a) Find $\mathbf{b} \times \mathbf{v}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) Find $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{v})$.

(c) Find the volume of the parallelepiped determined by $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{v}$.

THE UNIVERSITY OF MANITOBA

DATE: April 18, 2005 FINAL EXAMINATION

PAPER NO: 300 PAGE 7 of 8

DEPARTMENT & COURSE #: 136.130 TIME: 2 HOURS

EXAMINATION: Vector Geom. & Linear Algebra EXAMINERS: Various

---

[11] 8. Let $\mathbf{u} = (0, 0, 2, -1)$, $\mathbf{v} = (1, 0, 1, 2)$ and $\mathbf{w} = (1, -1, 0, 0)$ be vectors in R^4.

(a) Check if $\mathbf{u}$ and $\mathbf{v}$ are orthogonal or not.

(b) Check if $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ are linearly independent or not.

(c) Is $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ a basis for R^4 or not? Explain.

THE UNIVERSITY OF MANITOBA

DATE: April 18, 2005 FINAL EXAMINATION

PAPER NO: 300 PAGE 8 of 8

DEPARTMENT & COURSE #: 136.130 TIME: 2 HOURS

EXAMINATION: Vector Geom. & Linear Algebra EXAMINERS: Various

---

[13] 9. Let $V = M_{22}$ and let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

(a) Let $U = \{B \text{ in } M_{22} \text{ such that } AB = BA\}$.
Check if U is a subspace of V.

(b) Let $W = \{B \text{ in } M_{22} \text{ such that } B = A + a\,I, \quad a \text{ in } R\}$. Check if W is a subspace of V.

THE UNIVERSITY OF MANITOBA

DATE: April 18, 2005 FINAL EXAMINATION

PAPER NO: 300 PAGE 9 of 8

DEPARTMENT & COURSE #: 136.130 TIME: 2 HOURS

EXAMINATION: Vector Geom. & Linear Algebra EXAMINERS: Various

---

[16] 10. Given that the reduced row echelon form of the matrix

$$A = \begin{bmatrix} 3 & -3 & 0 & 21 & -6 \\ 1 & -1 & 1 & 2 & 1 \\ 2 & -2 & 0 & 14 & -4 \\ -2 & 2 & 0 & -14 & 4 \end{bmatrix} \text{ is } R = \begin{bmatrix} 1 & -1 & 0 & 7 & -2 \\ 0 & 0 & 1 & -5 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis of the column space of A.

(b) Find a basis of the row space of A.

(c) Find a basis of the null space of A.