THE UNIVERSITY OF MANITOBA

DATE: December 22, 2004 FINAL EXAMINATION

PAPER NO. 580

DEPARTMENT & COURSE NO: 136.130 TIME: 2 hours

EXAMINATION: Vector Geometry & Linear Algebra EXAMINERS: Various

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Values

[9] 1. (a) Suppose A is a 4×4 matrix with $\det A = 3$. Find $\det(2A)$.

(b) Suppose in addition that B is a 4×4 matrix with $\det B = 7$. Find $\det\left(A^T B\right)$.

(c) Find $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$.

(d) Let $\frac{1}{2}B^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Find B.

Values

[16] 2. (a) Find all values of the number *a* such that $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 0 & 2 \\ -2 & a & 2 \end{bmatrix}$ does not have an inverse.

In (b), (c) and (d), consider the matrix A as in part (a) and let $a=2$

(b) The adjoint of A is partially computed as shown. Fill in the three missing numbers in the boxes.

$$\text{Adj}(A) = \begin{bmatrix} -4 & \square & 4 \\ -8 & 4 & \square \\ \square & -8 & -4 \end{bmatrix}$$

(c) Find $\det A$.

(d) Find A^{-1}.

Values

[10] 3. Use Cramer's Rule to solve for only y: No other method will be awarded marks. Show all your work.

$$\begin{aligned} 2x + y - z &= -2 \\ 2x + y + 2z &= 1 \\ 2x - 2y + z &= -1 \end{aligned}$$

Values

[15] 4. Let $\mathbf{u} = (1, 1, 3)$ and $\mathbf{v} = (2, 2, -1)$. Find each of the following:

(a) The cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$;

(b) A non-zero vector orthogonal to both $\mathbf{u}$ and $\mathbf{v}$;

(c) The area of the parallelogram determined by $\mathbf{u}$ and $\mathbf{v}$.

Values

[15] 5. (a) Find parametric equations or the vector form for the line L passing through the points $P = (2,\ 4,\ -1)$ and $Q = (5,\ 0,\ 7)$.

(b) Find an equation of the plane passing through the point $(3,\ -1,\ 7)$ and perpendicular to the line $(x,\ y,\ z) = (7,\ 1,\ 2) + t(4,\ 2,\ -5)$.

Values

[9] 6. (a) Find the point P of intersection of the line $(x, y, z) = (2, 1, 3) + t(2, -2, 1)$ with the plane $x + y - z = 2$.

(b) How far is the point P from the origin?

Values

[20] 7. (a) Express (7, 3) as a linear combination of (2, 3) and (5, 6).

(b) Let U be the set of all vectors in $\mathbb{R}^3$ that are perpendicular to the vector (2, 3, 4). Give a reason why U is a subspace of $\mathbb{R}^3$.

(c) Find a basis for U of part (b).

(d) Let V be the following <u>subset</u> of $\mathbb{R}^3$: $V = \{(a, 5, b) \mid a, b \in \mathbb{R}\}$. Find a vector $\mathbf{v}$ so that $\mathbf{v} \in V$ but $2\mathbf{v} \notin V$.

(e) Let W be the following <u>subset</u> of $\mathbb{R}^2$: $W = \{(x, y) \mid x = 0 \text{ or } y = 0\}$: Find two vectors $\mathbf{u}$ and $\mathbf{w}$ in W so that $\mathbf{u} + \mathbf{w} \notin W$.

Values

[16] 8. Given: the reduced row echelon form of the matrix

$$A = \begin{bmatrix} 3 & 9 & -2 & 1 & 16 & 14 \\ 1 & 3 & -3 & 2 & 11 & 5 \\ 3 & 9 & 5 & -4 & -1 & 13 \\ 3 & 9 & -5 & 2 & 21 & 11 \end{bmatrix} \text{ is } R = \begin{bmatrix} 1 & 3 & 0 & 0 & 4 & 5 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Find a basis of the row space of A.

(b) Find a basis of the column space of A.

(c) Find a basis of the nullspace of A.

(d) The dimension of the row space of A is ______________________________.

The dimension of the column space of A is ___________________________.

The dimension of the nullspace of A is ______________________________.

Values

[10] 9. Let V be the vector space $M_{2,2}$ of all 2 x 2 matrices.

(a) Find a basis for V that contains $A = \begin{bmatrix} 2 & 3 \\ 5 & 2 \end{bmatrix}$ but does not contain $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

(b) Express $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ as a linear combination of the basis that you gave in part (a).