MATH1690, Assignment No. 1
September 19, 2007

The assignment is due Wednesday, September 26, in class. Late assignments receive a mark zero.

1. a) If x and y are rational numbers show that x+y and xy are also rational. [3]
   b) If x and y are irrational, must x+y and xy be also irrational? Explain.   [2]

2. Let f(x) = 3–|x+1| and g(x) = $\sqrt{2x+1}$.

   a) Find the domains and the ranges of f and g and draw their graphs. [4]
   b) Find the domains, ranges and formulas for the functions f∘g and g∘f (if the corresponding function is defined). Draw the graphs of f∘g and g∘f (if the corresponding function is defined).   [4]
   c) Check if f and g are one-to one, and if yes, find their inverse functions. [4]

3. a) The point P lies on the x axis and the point Q lies on the line y= –2x. The point (2, 1) is the midpoint of PQ. Find the coordinates of P. [4]
   b) Find the y coordinates of the points of intersection of the circle $x^2+y^2=4$ and the ellipse $x^2+2y^2+8y+4=0$. Draw a picture. [4]
   c) What is the length (no decimal numbers, please) of the line segment having one end on the x axis, the other end on the y axis, passing through the point $(1, 2-\frac{2}{\sqrt{3}})$ and making an angle of 150° with the positive part of the x axis. Draw a picture. [4]

4. A bacteria population doubles every four hours. If initially there are 100 bacteria:
   a) What is the size of the population after t hours?   [2]
   b) Find the inverse to the function in a) and explain its meaning. [4]
   c) When will the population reach 12 800? (No calculators; use only properties of logarithmic functions.)   [2]

5. Let $f(x) = x^2 + 2x - 1.5$.
   a) Knowing that the limit of f exists as x approaches –2, use tables to guess the value of the limit.   [2]
   b) Using the formal definition of the limit, verify your guess from a) for the value of $\lim_{x \to -2}(x^2 + 2x - 1.5)$.   [4]

Total [43/40]