MATH.1690, Assignment No. 4
January 23, 2008

The assignment is due Wednesday, January 30, 2008 in class. Late assignments receive a mark zero.

1. If a rectangle has its base on the x-axis and two vertices on the curve $y = e^{-x^2}$, show that the rectangle has the largest possible area when the two vertices are at the points of inflection of the curve. [8]

2. Let $a_1 = 3$, and let $a_{n+1} = 2a_n + 1$, for $n \geq 1$. Guess the formula for a_n in terms of n and prove it by using Mathematical Induction. [7]

3. Let $f(x) = \dfrac{1}{1+2x}$.

   a) Find the formula for $f^{(n)}(x)$, for all n in $\mathbb{N}$, by using Mathematical Induction. [4]

   b) Write the Taylor polynomial $P_n(x)$ and the Lagrange remainder $R_n(x)$ for f(x) around $x_0 = 0$. [4]

   c) Show that $\lim_{n\to\infty} |R_n(x)| = 0$, whenever 0< x < 1/2. [5]

4. Let f be a function such that $|f^{(n)}(x)| \leq 1, \forall n \in \mathbb{N}, \forall x \in \mathbb{R}$. Let $P_n(x)$ be the n-th Taylor polynomial for f around $x_0=0$.

   a) Estimate the error if $P_5(1/2)$ is used to approximate f(1/2). [3]

   b) Find the least n for which $P_n(-2)$ approximates f(–2) with an error smaller than 0.001. [4]

5. Let f be such that x+y = 0 is a tangent line to its graph at some point x_0, and such that the slope of the tangent line at a general point (x, f(x)) of the graph is x^3. Find f(x). [7]

Total [42/40]