MATH1690, Assignment No. 2

October 12, 2007

The assignment is due Friday, October 19, 2007 in class. Late assignments receive a mark zero.

1. Find the following limits. Show all of your work.

   a) $\lim_{x\to 0} \dfrac{x}{|x-1|-|x+1|}$ [2]

   b) $\lim_{x\to 2^-} \dfrac{x^2-4}{x^2-4x+4}$ [2]

   c) $\lim_{x\to -\infty} \dfrac{1}{\sqrt{x^2-4x}+x}$ [4]

   d) $\lim_{x\to 1}(f(x)\sin 3x+2)$, with f(x) such that $(\sin\frac{\pi}{2}x-1)\le f(x)\le (x-1)^2$. [4]

2. Using the formal definition for the limits, prove the following:

   a) $\lim_{x\to 1^-} \dfrac{1}{x-1} = -\infty$ [5]

   b) $\lim_{x\to -\infty} \dfrac{1}{\sqrt{x^2+1}} = 0$ [5]

3. a) Find the points of discontinuities for the function $f(x)=\dfrac{x+3}{|x|-3}$ and determine whether they are removable. Explain. [4]

   b) Give an example of two functions f and g that are not continuous at the point c, but such that (at the same time) f+g and fg are both continuous at c. [2]

4. If an odd function f is right continuous at x=0, show that it is continuous at x=0 and that f(0)=0. (Hint: prove first that $\lim_{x\to 0^-} f(x)$ exists and equals to $\lim_{x\to 0^+} f(-x)$.) [6]

5. a) Prove that if $f(x)\ge 0$ near a and $\lim_{x\to a} f(x)=L$, then $L\ge 0$. [6]

   b) If $f(x)>0$ near a and $\lim_{x\to a} f(x)=L$, must $L>0$? Why? [2]

Total [42/40]