MATH1690, Assignment No. 3
November 28, 2007

The assignment is due Wednesday, December 5, 2007 in class. Late assignments receive mark zero.

1.a) Show that for x > 0 we have that $\tan^{-1} x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$. [5]

b) Let f(x) = $\sin^{-1}(\cos x)$. What is the domain of f? Is f periodic? Where is f continuous? Where is f differentiable? Show that the derivative of f (where ever it exists) is either 1 or –1. Draw the graph of f. [7]

2. Find y' = $\frac{dy}{dx}$, given that:

a) $y = (x^3 - 5\tan^{-1}(1-x))^{\ln 3x}$. [3]

b) $\sin^{-1}(xy) = \cos^{-1}(x-y)$. [3]

3. Evaluate the limits :

a) $\lim_{x\to 0^+} \frac{\csc x}{\ln x}$. [3]

b) $\lim_{x\to 0}(1+\tan x)^{\frac{1}{x}}$. [4]

4. a) Let f'(x) = g'(x) for every x in $\mathbb{R}$, and let f and g have the same value at some point a in $\mathbb{R}$. Prove that then f(x) = g(x) for all x in $\mathbb{R}$.
(Hint: Look at h(x)=f(x)-g(x).) [4]

b) Show that if f is differentiable on an open interval I and f'(x) ≠ 0 on I, the equation f(x) = 0 can have at most one real root in I. (Hint: prove by contradiction plus the MVT.) [5]

5. Draw the graph of the function $f(x) = x + 1 - \frac{1}{x} - \frac{1}{x^2}$ by showing all the details of your work. [8]

Total [42/40]