

MATHEMATICS 136.169

Test 2
75 min.

February 25, 2003

Instructions: Attempt all questions. The total value of all questions is 62/60. Values of individual questions are printed beside the statement of the question. If you need more space use the reverse side of the page, but indicate clearly that you are doing so. There are two blank pages at the end of the test for you to use as scrap paper. Please fill in the information requested below. Good luck!

Name _____

Signature _____

Student No. _____

Question#	Mark
1	/11
2	/9
3	/9
4	/8
5	/7
6	/6
7	/12
TOTAL	/60

Value 1. a) Find $\lim_{x \rightarrow 0^+} \frac{\ln(\sin^{-1} x)}{\ln(x)}$

[5]

b) Find the slope of the tangent line to the curve $\tan^{-1}(2x) = \frac{\pi x}{y^2}$
at the point $(\frac{1}{2}, \sqrt{2})$.

[6]

2. a) Evaluate the integral $\int 2^x \cos(\pi 2^x) dx$

[3]

b) Find the area of the region enclosed by the graphs of $y = \frac{e^{-x}}{\sqrt{1-e^{-2x}}}$,
 $y = 0$, $x = \ln 2$ and $x = \ln \frac{2}{\sqrt{3}}$.

[6]

[3] 3. a) State the first part of the Fundamental Theorem of Calculus.

b) Find $\lim_{h \rightarrow 0} \frac{1}{h} \left(\int_0^{(1+h)^2} \frac{dt}{1+t^3} - \int_0^1 \frac{dt}{1+t^3} \right)$.

(Hint: use $F(x) = \int_0^{x^2} \frac{dt}{1+t^3}$ and explain why you can use part a.)

[6]

4. Find $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{\pi}{4n} \frac{1}{\cos^2\left(\frac{\pi k}{4n}\right)}$ by using Riemann sums over the interval

[8] $\left[0, \frac{\pi}{4}\right]$. Explain which theorem on integration you are using to get the answer.

5. Use Mathematical Induction to show that $\int_0^{2n\pi} |\sin x| dx = 4n$.

[7]

6. A rectangle is to be inscribed in a right triangle having sides of length 6 cm, 8 cm and 10 cm. Find the dimensions of the rectangle with greatest area if one corner of the rectangle overlaps with the right angle of the triangle and the opposite corner touches the hypotenuse.

[6]

7. Let $f(x) = e^{\frac{1}{x}}$. Then $f'(x) = \frac{-e^{\frac{1}{x}}}{x^2}$ and $f''(x) = \frac{e^{\frac{1}{x}}(1+2x)}{x^4}$.

- a) Find the domain of f , intercepts, symmetry and asymptotes (if any.)

[5]

- b) Find the intervals of monotonicity, critical and singular points.

[2]

- c) Determine concavity and inflection points.

[3]

- d) Sketch the graph of $f(x)$ by labeling the important points.

[2]