MATHEMATICS 136.169
EXAM
3 hours.
April 17, 2003

Instructions: Attempt all questions. The total value of all questions is 100. Values of individual questions are printed beside the statement of the question. If you need more space use the reverse side of the page, but indicate clearly that you are doing so. There are two blank pages at the end of the test for you to use as scrap paper.
Please fill in the information requested below. Good luck!

Name _____

Signature _____

Student No. _____

QUESTION#	MARK
1	/8
2	/8
3	/8
4	/7
5	/7
6	/11
7	/9
8	/10
9	/13
10	/12
11	/7
TOTAL	/100

Value

1. a) Write the $\varepsilon - \partial$ definition of $\lim_{x \to a} f(x) = L$.

[2]

b) **Using a),** prove your guess for the value of $\lim_{x \to 1}\left(\frac{x}{2} + 3\right)$.

[6]

2. Let $f(x) = \begin{cases} kx & ,x \le 2 \\ (x-2)^{(x-2)} & ,x > 2 \end{cases}$. Find the value of k such that f is continuous at x=2.

[8] (Hint: you will have to use L'Hopital's rule at one point.)

3. a) Write the definition of the derivative of a function.

[2]

b) Let u(x) = 3f(x) + g(x) . If f'(0)=1 and g'(0)= -1 prove by using a) that u'(0) = 2.

[6]

4. Write the equation of the tangent line to the graph of $y = (x + \cos^2 3x)^{\frac{1}{2}}$ at the point with x=0 .

[7]

5. Find the **dimensions** of the largest right-circular cylinder that can be inscribed in a sphere of radius R.

[7]

6. Let $f(x) = \frac{1}{x}$.

a) Guess the formula for $f^{(n)}(x)$ and then prove it by using Mathematical Induction.

[7]

b) Write the Taylor polynomial of degree n for f as above about x=1.

[4]

7.a) State the Mean Value Theorem for integrals.

[2]

b)Let f and g be two positive functions on the interval [a,b]. Let A_f (and A_g) be the areas under the graph of f(x) (and g(x)), above the x axis and between x=a and x=b. Prove that if $A_f = A_g$, then there exists a a point c in [a,b] such that f(c) = g(c) .
(Hint: Look at the function h=f-g and use a).)

[7]

8. a) State the first part of the Fundamental Theorem of Calculus.

[2]

-4-

b) Find $\lim_{h\to 0}\frac{1}{h}\left(\int_0^{(1+h)^2}\frac{dt}{1+t^3}-\int_0^1\frac{dt}{1+t^3}\right)$.

(Hint: use F(x) = $\int_0^{x^2}\frac{dt}{1+t^3}$ and explain why you can use part a).)

[6]

c) Is F (from b)) increasing or decreasing about x=1? Explain.

[2]

9. Evaluate the following integrals:

a) $\int_0^{\frac{\pi}{4}}\sec^2 x\tan^2 x\,dx$

[3]

b) $\int \cos^{-1} x\,dx$

[5]

c) $\int_1^{\sqrt{3}} \frac{1}{x^2\sqrt{4-x^2}}dx$.

[6]

10.a) Find the volume of the solid formed by the rotation of the graph of $f(x)=\frac{1}{\sqrt{x}\ln x}$ for x in (e,∞), about the x axis.

[6]

b)Let S be the solid formed when the graph of f(x)= $\frac{1}{x^2}$ for x in (0,1], rotates about the y axis. Show that S has infinite surface area.

[6]

11. Find the length of the curve $x(t) = a\cos^3 t$, $y(t) = a\sin^3 t$, $0 \le t \le \frac{\pi}{2}$.

[6]