MATHEMATICS 136.169
Test 1
75 min.
November 6, 2001

Instructions: Attempt all questions. The total value of all questions is 62/60. Values of individual questions are printed beside the statement of the question. If you need more space use the reverse side of the page, but indicate clearly that you are doing so. There are two blank pages at the end of the test for you to use as scrap paper.
Please fill in the information requested below. Good luck!

Name _____

Signature _____

Student No. _____

QUESTION#	MARK
1	/8
2	/10
3	/8
4	/8
5	/7
6	/8
7	/13
TOTAL	/60

Value

1. Find the following limits:

a) $\lim_{x\to 1} \dfrac{\sqrt{x} - 2x\sqrt{x} + x^2\sqrt{x}}{x-1}$

[2]

b) $\lim_{x\to -\infty} (\sqrt{x^6+5} + x^3)$

[2]

c) $\lim_{x\to 2^-} \dfrac{|x-3|+2x}{\sqrt{x-1}}$

[1]

d) $\lim_{x\to 0} \dfrac{2x^2}{1-\cos^2 x}$

[3]

2. Find the derivatives of the functions. Do not simplify.

a) $f(x) = (\sin\dfrac{\pi}{7})^2 + (\cos(5x^3))^4$

[3]

b) $f(x) = \dfrac{\tan x + \sqrt{2x}}{5x - 3x^2}$

[4]

c) $f(x) = \sqrt[3]{2x^2 + (x-1)^3 \sin(2-x)}$

[3]

3. a) Write the formal definition for $\lim_{x \to a} f(x) = L$.

[2]

b) Prove that $\lim_{x \to -3} \frac{x^2 - 9}{2(x+3)} = -3$ by using the formal definition of the limit.

[6]

4. a) Write the definition of when is f continuous at the point c.

[2]

b) For x ≥ -1, let $f(x) = \frac{1}{|x|} - \sqrt{\frac{x+1}{x^2}}$. Check if the discontinuity at x = 0 is removable or not.
(Hint: $\sqrt{x^2} = ?$.)

[6]

5. a) Write the definition of the derivative of f at the point c.

[2]

b) If it is given that the function f is differentiable at x=c, and that the function g is given by g(x) = 2f(x) +3, prove that g is differentiable at

[5] c by using the formal definition of the derivative? What is the derivative of g at c equal to?

6. a) Write the equation of the tangent line to the curve y=f(x) at the

[2] point (x_0, y_0).

b) Find all values of k such that the graph of f(x) = $\frac{x^3}{3} + k$ has

[6] tangent line y = 2x. (Hint: first find the points $(x_0, f(x_0))$ on the curve at which the slope of the tangent line is 2.)

7. a) State the Intermediate value theorem.
[3]

b) State the Min.-Max. Theorem.
[3]

d) If f is continuous on [a,b], show that the range of f is a closed interval
[7] [c, d] .
(Hint: be careful, you have the prove that the whole interval [c,d] is covered.)