

136.169, Assignment No. 2

October 14, 2005

The assignment is due Friday, October 21, 2005 in class. Late assignments receive a mark zero.

1. Find the following limits. Show all of your work.

a) $\lim_{x \rightarrow 0} x \cos \frac{3-x}{x}$. [3]

b) $\lim_{x \rightarrow -\infty} \frac{1}{\sqrt{x^2 - 4x} + x}$. [4]

c) $\lim_{x \rightarrow 2} f(x)$ if $\lim_{x \rightarrow 2} \frac{f(x) - 3}{x} = -1$. [3]

2. a) Find the points of discontinuities for the function $f(x) = \frac{1+x}{1-|x|}$, and determine if they are removable or not. [6]

b) Find the value for the constant k such that the function $f(x) = \begin{cases} 3k + x, & x \leq -1 \\ 1 - kx^2, & x > -1 \end{cases}$ is continuous at $x = -1$. [5]

c) Give an example of two functions f and g that are not continuous at the point c , but such that $f+g$ and $f \cdot g$ are both continuous at c . [3]

3. Let $K > 0$ and let f be defined on all of \mathbb{R} . Show that if f satisfies the condition $|f(x) - f(y)| \leq K|x-y|$, for all x and y in \mathbb{R} , then f is continuous at every point c in \mathbb{R} . [7]

4. Let $f(x) = \sqrt{1-3x}$.

a) Find $f'(x)$ by using the definition of the derivative. [5]

b) Find the domains of f and f' . [2]

c) Find the equation of the tangent line to the graph of f at $x = -1$. [3]

Total [41/40]