136.169, Assignment No. 1

September 23, 2005

The assignment is due Friday, September 30, in class. Late assignments receive a mark zero.

1. Let f(x) = 3–|x| and g(x) = $\sqrt{x-2}$.

   a) Find the domains and the ranges of f and g and draw their graphs. [4]

   b) Find the domains, ranges and formulas for the functions $f \circ g$ and $g \circ f$ (if the corresponding function is defined). Draw the graphs of $f \circ g$ and $g \circ f$ (if the corresponding function is defined).[5]

   c) Check if f and g are one-to one, and if yes, find their inverse functions. [4]

2. a) The point P lies on the x axis and the point Q lies on the line y = –2x. The point (2,1) is the midpoint of PQ. Find the coordinates of P. [4]

   b) Find the y coordinates of the points of intersection of the circle $x^2+y^2=4$ and the ellipse $x^2+2y^2+8y+4=0$. Draw a picture. [4]

   c) What is the length (no decimal numbers, please) of the line segment having one end on the x axis, the other end on the y axis, passing through the point $(1, 2-\frac{2}{\sqrt{3}})$ and making an angle of 150° with the positive part of the x axis. Draw a picture. [4]

3.Using the formal definition of the limit, verify that:

   a) $\lim_{x\to -2}(5-2x-x^2)=5$. [4]

   b) If $\lim_{x\to 2} f(x)=L, L>0$, show that there exists $\delta>0$ such that, whenever $0<|x-2|<\delta$, we have that $f(x)>0$.[6]

4.Showing all of your work, find:

   a) $\lim_{x\to 1}\left(\left(\frac{3-x^2}{x^2-2x-1}\right)^3-2x\right)$. [3]

   b) $\lim_{x\to 2^-}\frac{2x^3-4x^2}{|2x-4|}$. [3]

Total [41/40]