136.169, Assignment No. 4
January 20, 2006

The assignment is due Friday, January 27, 2006 in class. Late assignments receive a mark zero.

1. Find the n-th derivative of f(x) = ln (x+1). Prove the formula by using induction.[5]

2. Let f(x) = ln (x+1).

   a)Write the Taylor polynomial Pn(x) and the Lagrange remainder Rn(x) for f(x) around $x_0 = 0$. [3]

   b) Show that $\lim_{n\to\infty} |R_n(x)| = 0$ for x=1. [5]

   c) Find n such that the approximation of ln2 by the n-th Taylor polynomial Pn for ln (x+1) around 0 is with an error smaller than 0.001. [5]

3. Evaluate the upper and the lower Riemann sums U(f, Pn) and L(f, Pn) for f(x)= 1/x on the interval [1, 2], for the partition Pn with division points $x_i = 2^{\frac{i}{n}}$, for 0≤ i≤ n. Verify that $\lim_{n\to\infty} U(f, Pn) = \ln 2 = \lim_{n\to\infty} L(f, Pn)$. Explain why you can conclude that it must be that $\int_1^2 \frac{1}{x} dx = \ln 2$. [10]

4. Using the properties of integrals and the fact that $\int_0^{\frac{\pi}{2}} \cos x dx = 1$, find the value of $\int_0^{\frac{\pi}{2}} (2\cos x - 5x) dx$. [6]

5. Let f and g be two positive functions on the interval [a, b]. Let A_f (and A_g) be the area under the graph of f(x) (and g(x)), above the x axis and between x=a and x=b. Prove that if $A_f = A_g$, then there exists a point c in [a, b] such that f(c) = g(c) . [6]

Total [40]