

136.169, Assignment No. 3

November 28, 2005

The assignment is due Monday, December 5, 2003 in class. Late assignments receive mark zero.

- 1.a) Show that for $x > 0$ we have that $\tan^{-1} x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$. [5]
- b) Let $f(x) = \sin^{-1}(\cos x)$. What is the domain of f ? Is f periodic? Where is f continuous? Where is f differentiable? Show that the derivative of f (where ever it exists) is either 1 or -1 . Draw the graph of f . [7]
2. Evaluate the limits :
- a) $\lim_{x \rightarrow 0^+} \frac{\csc x}{\ln x}$. [3]
- b) $\lim_{x \rightarrow 0} (1 + \tan x)^{\frac{1}{x}}$. [3]
- c) $\lim_{x \rightarrow 1^+} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$. [3]
3. a) If $f(x) = \frac{\ln x}{x}$, show that $f'(x) > 0$ on $(0, e)$ and $f'(x) < 0$ on (e, ∞) . What is the value of f at $x=e$? Use the above to show that $e^\pi > \pi^e$. [4]
- b) Prove that $x^2+1 - \ln x > 0$ on $(0, \infty)$. [3]
4. Draw the graph of the function $f(x) = x + 1 - \frac{1}{x} - \frac{1}{x^2}$ by showing all the details of your work. [8]
5. You are riding on a Ferris wheel of diameter 20m. The wheel is rotating at 1 revolution per minute. How fast are you rising or falling when you are 6m horizontally away from the vertical line passing through the center of the wheel? [6]

Total [42/40]