Prove the following statements by using Mathematical Induction:

1. $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$, for all n in $\mathbb{N}$.

2. If r$\neq$ 1, then $1 + r + r^2 + ... + r^n = \frac{1 - r^{n+1}}{1 - r}$, for all n in $\mathbb{N}$.

3. $5^{2n} - 1$ is divisible by 6, for all n in $\mathbb{N}$.

4. $n < 2^n$, for all n in $\mathbb{N}$.

5. $2^n < n!$, for all n$\geq$4.

6. $a - b$ is a factor of $a^n - b^n$, for all n in $\mathbb{N}$.

7. Let $a_1 = 3$, and let $a_{n+1} = 2a_n + 1$, for $n \geq 1$. Find the formula for $a_n$ in terms of $n$.

8.

$$1=1$$
$$1-4 = -(1+2)$$
$$1-4+9 = 1+2+3$$
$$1-4+9-16 = -(1+2+3+4)$$
$$\ldots\ldots\ldots$$

Guess the general formula suggested and prove it by using Mathematical Induction.

9. Prove that for every $x \geq 0$, and every n in $\mathbb{N}$, $e^x \geq 1 + x + \frac{x^2}{2!} + ... + \frac{x^n}{n!}$.

10. Let $\binom{n}{m} = \frac{n!}{(n-m)!m!}$, for n and m non-negative integers, and let *a* and *b* be in $\mathbb{R}$.

Prove that $\binom{n}{m} + \binom{n}{m-1} = \binom{n+1}{m}$, for $1 \leq m \leq n$, and use it to prove that

$$(a+b)^n = \sum_{m=0}^{n} \binom{n}{m} a^{n-m} b^m \text{, for all n in } \mathbb{N}.$$