MATH 2200
Test 1

October 20, 2006

Instructions: Attempt all questions. The total value of all questions is 39(+2). Values of individual questions are printed beside the statement of the question. If you need more space use the reverse side of the page, but indicate clearly that you are doing so. There are two blank pages at the end of the test for you to use as scrap paper.
Please fill in the information requested below.

Name (print)___________________________

Student No.____________________________

Signature______________________________

QUESTION#	MARK
1	/6
2	/7
3	/8
4	/10
5	/8
TOTAL	/39

Value

1. Define the number 2 by $2 = 1+1$.

[4] a) Prove , by mentioning which field axioms you are using in each step, that $2 \neq 1$. (Hint: use proof by contradiction.)

[2] b) Prove, by mentioning which order axioms and theorems you are using in each step, that $2 \neq 0$. (Hint: prove that $2 > 0$. No proof by contrad. here.)

2. Let $f: A \to B$ and let G and H be subsets of B.

[1] a) State the definition of $f^{-1}(G)$.

[3] b) Prove that if $G \subseteq H$, then $f^{-1}(G) \subseteq f^{-1}(H)$.

[3] c) If $G \subseteq H$ and $f^{-1}(G) = f^{-1}(H)$, must $G = H$? Prove your answer.

3. Let $I_r = (r, 1)$, for r in $(0, 1) \cap \mathbb{Q}$.

a) State the density theorem (for rational numbers).

[2]

b) If $A = \bigcup_{r \in (0,1) \cap \mathbb{Q}} I_r$, prove that $A = (0, 1)$. (Use a) in one half of the proof.)

[6]

4. a) Prove, by using the Principal of Mathematical Induction, that $\frac{1}{2^n} < \frac{1}{n}$,

for every n in $\mathbb{N}$.

[4]

b) Define the infimum of a set A that is bounded from below.
[2]

c) Show that the infimum of the set A = $\{\frac{1}{2^n} : n \in \mathbb{N}\}$ is 0. (Hint: use a corollary of the Archimedean property and part a).)

[4]

d) **BONUS QUESTION**: Find the set A = $\bigcap_{n=1}^{\infty} I_n$, with $I_n = [-1, \frac{1}{2^n}]$, by
[2] using c) and a consequence of the proof of the Nested Intervals Theorem as stated in class.

5. a) Define when is a function f: A $\rightarrow$B an injection and when is it a surjection.

[2]

b) Prove in details that the set A = $\{\frac{m^2+1}{3} : m \in$ {even natural numbers}} is denumerable (i.e. infinite countable). (You have to find a bijection f between $\mathbb{N}$ and A and show me that that f is a bijection.)

[6]