MATH 2200, Assignment No. 3
November 8, 2006

The assignment is due Wednesday, November 15, in class. Late assignments receive a mark zero.

1. a) Using the **definition of the limit** of a sequence prove that $\lim_{n\to\infty} \frac{\sqrt{n}}{n+1} = 0$. [5]

   b) Prove that if lim $x_n = 0$ and the sequence (y_n) is bounded, then lim $x_n y_n = 0$. [5]

   c) Give examples of sequences (x_n) and (y_n) such that lim $x_n = x \neq 0$, (y_n) is bounded , but lim $x_n y_n$ does not exist. [2]

2. Determine if (x_n) converges or diverges: (**Explain** why.)

   a) $x_n = \frac{\sin 2n}{n} + \frac{n^2}{1-n^2}$. [4]

   b) $x_n = (-1)^n \sqrt{n}(\sqrt{n+1} - \sqrt{n})$. [6]

   c) $x_n = \frac{2^n}{n}$. (What is $\lim_{n\to\infty} \frac{x_{n+1}}{x_n}$ equal to? Use this to prove your claim.) [7]

3. a) Let (x_n) be a bounded sequence and let s= sup{ x_n : n in IN }. Show that if s is not in { x_n : n in IN }, then there is a subsequence of (x_n) that converges to s. [7]

   b) Show that if (x_n) is unbounded, then there exists a subsequence (x_{n_k}) such that $\lim_{k\to\infty} \frac{5}{x_{n_k}} = 0$. [6]

4. a) Show, by using the **definition of a Cauchy sequence**, that if (x_n) and (y_n) are Cauchy sequences, then (x_n+y_n) and $(x_n y_n)$ are also Cauchy sequences. [7]

   b) Let (x_n) be a Cauchy sequence such that x_n is an integer, for all n in IN. Show that (x_n) is ultimately constant. [6]

Total [55/54]