MATH 2200, Assignment No. 4
November 24 , 2006

The assignment is due Friday, December 1, 2006 in class. Late assignments receive a mark zero.

1. a) Show that $\lim_{x \to -1} \frac{x+5}{2x+3} = 4$, by using the definition of a limit. [6]

   b) Show that $\lim_{x \to 1} \frac{1}{1-x}$ does not exist. [4]

2. a) Define g: $\mathbb{R} \to \mathbb{R}$ by g(x) = 3x for x rational and g(x) = x-2 for x irrational. Find all the points at which g is continuous. Prove your statements. [6]

   b) Let f: $A \to \mathbb{R}$ be nonnegative on A, and let f be continuous at c. Prove by using the definition of limit that then $\lim_{x \to c} \sqrt{f(x)} = \sqrt{\lim_{x \to c} f(x)}$, and so the function $\sqrt{f}$ is also continuous at c. [6]

3. a) Let f: $\mathbb{R} \to \mathbb{R}$ be continuous at c and let f(c) > 0. Show that there exists a neighbourhood $V_\delta(c)$ of c such that for any x in $V_\delta(c)$ we have that f(x) > 0.[6]

   b) Let f: $\mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$ and such that $f(m / 2^n) = 0$ for all m in $\mathbb{Z}$ and all n in $\mathbb{N}$. Show that f(x) = 0, for all x in $\mathbb{R}$. [7]

   c) Let f: $\mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$ and let S= {x in $\mathbb{R}$: f(x) = 0}. If (x_n) is contained in S and x = lim (x_n) , show that x is also in S. [3]

4. a) Give an example of functions f and g that are both discontinuous at a point c in $\mathbb{R}$ but such that both f+g and fg is continuous at c. [3]

   b) Give an example of a function f that is discontinuous at every point of $\mathbb{R}$, but such that $| f |$ is continuous on $\mathbb{R}$. [2]

   c) Show that if f and g are such that f is continuous at c, g is discontinuous at c and fg is continuous at c, then f must be zero at c . [7]

Total [50/48]