MATH 2200, Assignment No. 1

September 22, 2006

The assignment is due Friday, September 29, 2006 in class. Late assignments receive a mark zero.

1. Given sets A and B, show that :

   a) The sets $A \cap B$ and $A \setminus B$ are disjoint. [5]

   b) $A = (A \cap B) \cup (A \setminus B)$. [5]

2. a) Let $f(x) = -x^2$ and $g(x) = \frac{1}{\sqrt{x}}$. Find the functions $f \circ g$ and $g \circ f$ if they exist, by first finding the domains, codomains and ranges of f and g. Draw the graphs of all of the functions that do exist. [7]

   b) Show that if $f: A \to B$ is surjective and $H \subseteq B$, then $f(f^{-1}(H)) = H$. Give an example to show that the equality need not hold if f is not surjective. [7]

   c) If f is a bijection of A onto B, show that f^{-1} is a bijection of B onto A. [5]

3. Prove (by referring to each Field axiom and each Theorem as you are using it) that:

   a) 2+2 = 4, using that a natural number $n \neq 1$ is defined as $(n-1) + 1$. [3]

   b) If $a \neq 0$ and $b \neq 0$, show that $(ab)^{-1} = a^{-1}b^{-1}$. [6]

   c) For a not zero, $(-a)^{-1} = -(a^{-1})$, i.e. the inverse of a negative is the negative of the inverse. (Caution: do not write 1/a instead of a^{-1}.) [8]

4. For a and b in $\mathbb{R}$ let $a \# b := \frac{ab}{2} + 1$. (Note: a # b is a binary operation mapping $\mathbb{R} \times \mathbb{R} \to \mathbb{R}$. Check if for all a, b and c in $\mathbb{R}$:

   a) $a \# b = b \# a$, [2]

   b) $a \# (b \# c) = (a \# b) \# c$, [4]

   c) there exists e in $\mathbb{R}$ such that , for all a in $\mathbb{R}$, $a \# e = e \# a = a$. [4]

Total [56/54]