MATH 2202, Assignment No. 4
November 27, 2009

The assignment is due Friday, December 4, 2009 in class. Late assignments receive a mark zero.

1. a) Show that $\lim_{x\to -1}\frac{x-5}{2x+3} = -6$, by using the definition of a limit. [6]

   b) Show that $\lim_{x\to 3}\frac{1}{3-x}$ does not exist, by showing that f(x)= $\frac{1}{3-x}$ is not bounded in any delta neighbourhood of 3. [5]

2. a) Define g: $\mathbb{R} \to \mathbb{R}$ by g(x) = 2x for x rational and g(x) = x– 4 for x irrational. Find all the points at which g is continuous. Prove all of your statements. [7]

   b) Let f: A$\to \mathbb{R}$ be nonnegative on A, and let f be continuous at c. Prove by using the definition of limit that then $\lim_{x\to c}\sqrt{f(x)} = \sqrt{\lim_{x\to c} f(x)}$, and so the function $\sqrt{f}$ is also continuous at c. [7]

3. a) Let f: $\mathbb{R} \to \mathbb{R}$ be continuous at c and let f(c) > 0. Show that there exists a neighbourhood $V_\delta(c)$ of c such that for any x in $V_\delta(c)$ we have that f(x) > 0.[7]

   b) Let f: $\mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$ and such that f(m / 2^n) = 0 for all m in $\mathbb{Z}$ and all n in $\mathbb{N}$. Show that f(x) = 0, for all x in $\mathbb{R}$. (Hint: show first that $\forall x > 0, \forall \varepsilon > 0$ there exist n, m in $\mathbb{N}$ such that $\left| x - \frac{m}{2^n} \right| < \varepsilon$.) [7]

   c) Let f: $\mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$ and let S= {x in $\mathbb{R}$: f(x) = 0}. If (x_n) is contained in S and x = lim (x_n), show that x is also in S. [3]

4. a) Give an example of functions f and g that are both discontinuous at a point c in $\mathbb{R}$ but such that both f+g and fg is continuous at c. [3]

   b) Give an example of a function f that is discontinuous at every point of $\mathbb{R}$, but such that | f | is continuous on $\mathbb{R}$. [2]

   c) Show that if f and g are such that f is continuous at c, g is discontinuous at c and fg is continuous at c, then f must be zero at c . [4]

5. Determine all of the points of continuity of $f(x) = \frac{3x^2 + \sin\frac{1}{x}}{1-\sqrt{5-|x+3|}}$. Prove your statements by using (and stating) at each step the relevant theorems on combinations of continuous functions. [5]

Total [56/54]