

MATH 2202, Assignment No. 1

September 23, 2009

The assignment is due Wednesday, September 30, 2009 in class. Late assignments receive a mark zero.

1. a) Prove that if $C=A \cup B$ and $A \cap B = \emptyset$, then $A=C \setminus B$. [5]

b) Let $A_n = \left[0, 2 - \frac{1}{n}\right)$, with $n \in \mathbb{N}$. Show that $\bigcup_{n=1}^{\infty} A_n \subseteq [0, 2)$ and that $[0, 1) \subseteq \bigcap_{n=1}^{\infty} A_n$.

Does $2 \in \bigcup_{n=1}^{\infty} A_n$ and does $1 \in \bigcap_{n=1}^{\infty} A_n$? Explain. [8]

2. Let $f(x) = \sqrt{1-x}$ and $g(x) = x^2 - 1$. Find the functions $f \circ g$ and $g \circ f$ if they exist, by first finding the domains, co-domains and ranges of f and g . Draw the graphs of all of the functions that do exist (including f and g). [8]

3. a) Show that if $f: A \rightarrow B$ is surjective and $H \subseteq B$, then $f(f^{-1}(H)) = H$. Give an example to show that the equality need not hold if f is not surjective. [6]

b) If f is a bijection (from A onto B), show that f^{-1} is a bijection (from B onto A). [6]

4. a) Find a bijection from \mathbb{N} into all odd integers greater than 13. [3]

b) Find an example of a countable collection of finite sets, such that the union is not finite, and a countable union of finite sets such that the union is finite. [4]

c) Let A be countable infinite and let $A \subseteq B$. Prove that B is infinite. (You can use Theorem 1.3.3. that says that \mathbb{N} is an infinite set and any other theorem proven in class.) [6]

5. For a and b in \mathbb{R} let $a \# b := \frac{ab}{2} + 1$. (Note: $a \# b$ is a binary operation mapping

$\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$.) Check if for all a, b and c in \mathbb{R} :

a) $a \# b = b \# a$, [2]

b) $a \# (b \# c) = (a \# b) \# c$, [4]

c) there exists e in \mathbb{R} such that, for all a in \mathbb{R} , $a \# e = e \# a = a$. (Note: the question asks if there is one e that would do for all of the a 's.) [4]

Total [56/54]