MATH 2202, Fall 2008
Test 1, 1h

October 22, 2008

Instructions: Attempt all questions. The total value of all questions is 52 (+4 bonus). Values of individual questions are printed beside the statement of the question. If you need more space use the reverse side of the page, but indicate clearly that you are doing so. There are two blank pages at the end of the test for you to use as scrap paper.
Please fill in the information requested below.

Name (print)__________________________

Student No.__________________________

Signature____________________________

QUESTION#	MARK
1	/11
2	/8
3	/10
4	/12
5	/11
TOTAL	/52

Value

1. Let $f: A \rightarrow B$ and let X and Y be subsets of A.

[3] a) Prove that if $X \subseteq Y$, then $f(X) \subseteq f(Y)$.

[3] b) Show that the converse of the statement in a) is false, i.e. give an example of f, A, B, X and Y (subsets of A) such that $f(X) \subseteq f(Y)$, but X is not a subset of Y.

[5] c) Show that if f is an injection, then $f(X) \subseteq f(Y)$ implies that $X \subseteq Y$. **State** first the definition of an injection.

2. Prove the following by using the Field and Order axioms.

[5] a) Show that if $a > 0$, then $a^{-1} > 0$ too. (You can use Theorems proven in class such as $0x=0$, $-x = (-1)x$ and $1 > 0$.)

[3] b) $1^{-1} = 1$. (You can use the Theorem on uniqueness of the inverse.)

3.. a) Define the infimum of a set A that is bounded from below.
[3]

[7] b) Let $A = \{ x : |x-1| < x \}$. Find the interval form of A and the infimum and the supremum of A if they exist. Use a) and show all of your work.

Bonus question: c) Define a supremum of a set A that is bounded above and prove that if A and B are nonempty subsets of $\mathbb{R}$, B is bounded and $A \subseteq B$, then A is also bounded and $\sup A \leq \sup B$.
[4]

4. a) State the Nested Intervals Theorem..
[2]

b) State the Archimedean Property.
[2]

c) Let $I_n = \left(0, \frac{1}{n}\right]$. Prove that $\bigcap_{n=1}^{\infty} I_n = \varnothing$, by using b). (Hint:Use proof
[6] by contradiction.)

d) Does c) contradict a) ? Explain why.
[2]

5. Let (x_n) be a sequence of real numbers.

a) State the definition of $\lim_{n\to\infty} x_n = x$.

[2]

b) Prove, by using the definition of limit, that for any b in $\mathbb{R}$, $\lim_{n\to\infty} \frac{b}{n} = 0$.

[5]

c) Show that $\lim_{n\to\infty} \frac{2\cos n^3}{n} = 0$, by using b). (You can also use any theorem on limits proven in class, and the properties of the cosx function. State all theorems that you are using.)

[4]