MATHEMATICS 136.220
Test 1

October 20, 2000

Instructions: Attempt all questions. The total value of all questions is 53. Values of individual questions are printed beside the statement of the question. If you need more space use the reverse side of the page, but indicate clearly that you are doing so. There are two blank pages at the end of the test for you to use as scrap paper.
Please fill in the information requested below.

QUESTION#	MARK
1	/9
2	/11
3	/11
4	/10
5	/12
TOTAL	/53

Name (print)_____

Student No._____

Signature_____

Value

1. a) Let A and B be sets. Define the sets $A \cap B$ and A\B.

[3]

b) Prove that A\B = A if and only if . $A \cap B = \emptyset$.
   (Hint: Use prove by contradiction in one half of the proof.Think which half
   should it be ?)

[6]

2. a) Define when is a function f: $A \rightarrow B$ and injection and when is it a surjection.

[3]

b) Define when is a set A countable.

[2]

c) Prove in details that the set A = { 3n+1 : n in |N } is countable.
(In your proof reffer to at least two field axioms that you are using when manipulating the equations mentioned.)

[6]

3. In the parts a) and b) below refer to at lesat two order axioms whenever you are using them. You can use theorems from the "fields section" and from the 'order section".

a) Show that if $a > 0$, then $a^{-1} > 0$ too.

[5]

b) Use induction to prove that $n^2 \geq n$ for all n in |N.

[6]

4. a) Define the infimum of a set A that is bounded from below.

[3]

b) Let $A = \{ x : |x-1| \leq x \}$. Find the interval form of A and the infimum and the supremum of A if they exist. Use a) and show all of your work.

[7]

5. a) State the Nested Intervals Theorem..

[2]

b) State the Archemidean Property.

[2]

c) Let $I_n = (0, \frac{1}{n}]$. Prove that $\bigcap_{n=1}^{\infty} I_n = \varnothing$ by using b). (Hint:Use proof

[6]   by contradiction.

d) Does c) contradict a) ? Explain why.

[2]