MATHEMATICS 237
Test 1

November 17, 1999

Value

1. Let f: A $\rightarrow$ B and let X and Y be subsets of A.

[3] a) Prove that if $X \subseteq Y$, then $f(X) \subseteq f(Y)$.

[3] b) State the converse of the statement in a) and show that it is false.

[3] c) Show that if f is an injection, then $f(X) \subseteq f(Y)$ implies that $X \subseteq Y$.

2. Using the Field axioms and the uniqueness of the negative and the inverse prove that:

[4] a) $-a - b = -(a+b)$, for every a and b in $\mathbb{R}$.

[2] b) $1^{-1} = 1$.

3. a) Define the supremum of a nonempty bounded subset of $\mathbb{R}$.
[3]

b) Prove that if A and B are nonempty subsets of $\mathbb{R}$, B is bounded and $A \subseteq B$, then A is also bounded and $\sup A \leq \sup B$.
[4]

4. a) Define the limit x of a sequence (x_n) of real numbers.
[2]

b) State the Archimedean Property.
[2]

c) Prove that the definition in a) is equivalent to the following statement:

$$\forall k \in N, \exists N(k) \in N \text{ such that } \forall n \geq N \text{ we have that } |x_n - x| < \frac{1}{k}.$$

[5]

5. a) State the Monotone Convergence Theorem .
[2]

b) Prove by induction that the sequence (x_n) with $x_n = \frac{n}{2^n}$ is monotonic decreasing.
[4]

c) Prove that the sequence in b) converges, by using a). What is its limit ?
[2]

6. a) State the Bolzano-Weierstrass theorem.
[2]

c) Prove that a bounded sequence that does not converge has more than one accumulation points.
[6]